Derivatives, Slopes, Tangent Lines, and Making Movies

Introduction

OBJECTIVE: To visualize the derivative and the linearization of a function at a point.

In this module, we explore the derivative as the slope of a nonlinear function and find the equation of the line tangent to a curve at a point. You will learn how to plot the curve and selected tangents on the same graph. In addition, you will see how to use *Mathematica* to make a movie animation by generating a sequence of plots, each showing a different tangent to the curve. When the sequence of graphs is animated, the tangent lines appear to roll along the graph of the function.

Technology Guidelines

NOTE: If you have just finished a module, restart *Mathematica* or close the *Kernel* before executing a new module.
TO OPEN CELLS, put your cursor on the right cell bracket and double click.
INITIALIZATION CELLS
When asked if you want to ". . . automatically evaluate all the initialization cells in the notebook . . . ," respond by pressing the "Yes" button.
TO STOP AN EXECUTION
Select the *Kernel* pull-down menu and click on *Abort Evaluation.*
ORDER OF EXECUTION
Execute cells in the order given. Do not skip any Input cells within a given notebook.
SAVING NOTEBOOKS
You can save anytime to any directory you choose, and it is wise to save often.
However, before you do your final save, delete all your output by selecting the *Delete All Output* selection under the *Kernel* pull-down menu.
EXPERIENCING MAJOR PROBLEMS
Save if appropriate, then shut down *Mathematica* and start it up again.

Part I: The Derivative at a Point

Chapter 3, Section 1

Define a nonlinear function of your choice, and call it y=f(x) and graph it. For an example, we choose the function $f(x)=x^2$ for $-2\leq x\leq 2$. (To enter a different function and domain, change the red entries in the following input cell.)

In[212]:=

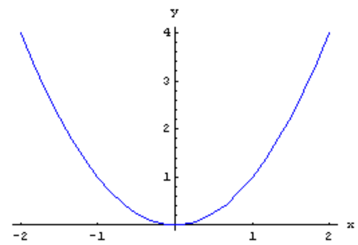
```
Clear[y, f];

x0 = -2;  xf = 2;

y = f[x_] = x^2
```

Out[214]=

x^2

In[215]:=

```
Plot[y, {x, x0, xf}, PlotRange → All,
 PlotStyle → {RGBColor[0, 0, 1]},
 AxesLabel → {"x", "y"}];
```



Now pick a point on the function and *use the definition of the derivative* to find the slope of the function's graph at the point you pick, calling it $m_{tangent}$. We choose x=1 for our example.

In[216]:=

$$m_{tangent} = \text{Limit}\left[\frac{f[1+h]-f[1]}{h}, h \to 0\right]$$

Out[216]=

2

Part II: The Linearization of a Function

Chapter 3, Sections 1 and 8 (ET Sections 1 and 10)

Form a new function for the line that is tangent to the function that you chose in the "You Try It" section at the point you picked in Part I, and call it $y_{tangent}$ **=L[x_]**. This function is called the *linearization* of y=f(x) at the point (1, f(1))

In[217]:=

$$y_{tangent} = L[x\_] = f[1] + m_{tangent} * (x - 1)$$

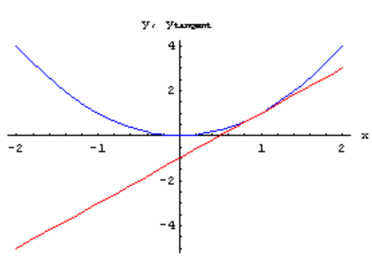
Out[217]=

1 + 2 (-1 + x)

Plot the function and the tangent line on the same graph.

In[218]:=

```
Plot[{f[x], L[x]}, {x, x0, xf},
 PlotRange → All,
 PlotStyle → {{RGBColor[0, 0, 1]},
   {RGBColor[1, 0, 0]}},
 AxesLabel → {"x", "y, ytangent"}];
```



Part III: The Derivative Function

Chapter 3, Section 1

Form the derivative function that will give the slope of the tangent to your chosen function at any point with coordinates (x,f(x)). Call the derivative function **yprime[x_]** , and graph it.

In[219]:=

```
yprime[x_] = Limit[(f[x + h] - f[x])/h, h → 0]
```

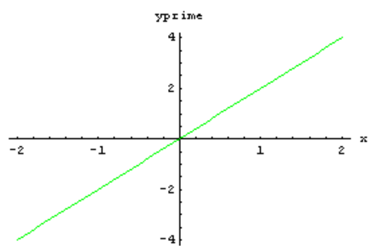
Out[219]=

```
2 x
```

Graph **yprime[x]**.

In[220]:=

```
Plot[yprime[x], {x, x0, xf},
 PlotStyle → {RGBColor[0, 1, 0]},
 AxesLabel → {"x", "yprime"}];
```



Part IV: A Whole Bunch of Tangents

Chapter 3, Section 1

Form a new *Mathematica* function that gives the equation of the line tangent to y=f(x) at the point (a,f(a)). Call the new function **tanline[x_, a_]**.

In[221]:=

```
Clear[tanline];

tanline[x_, a_] := f[a] + yprime[a] * (x - a)
```

Test your **tanline[x_,a_]** for several values of *a* by plotting the tangent lines and y=f(x) together on the same graph. First, use the **tanline[x_, a_]** function and the **Table[ ]** command to generate a list of equations for the tangents to the curve at points (a,f(a)), for values of *a* varying from -2 to 2 in increments of 0.1. Then graph the tangent lines and y=f(x) together on the same graph.

In[223]:=

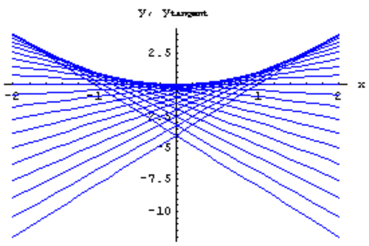
```
listoftangentlines =
 Table[tanline[x, a], {a, x0, xf, 0.2}]
```

Out[223]=

```
{4 - 4 (2 + x), 3.24 - 3.6 (1.8 + x), 2.56 - 3.2 (1.6 + x), 1.96 - 2.8 (1.4 + x), 1.44 - 2.4 (1.2 + x),
 1. - 2. (1. + x), 0.64 - 1.6 (0.8 + x), 0.36 - 1.2 (0.6 + x), 0.16 - 0.8 (0.4 + x), 0.04 - 0.4 (0.2 + x),
 1.2326 × 10^-32 + 2.22045 × 10^-16 (-1.11022 × 10^-16 + x), 0.04 + 0.4 (-0.2 + x), 0.16 + 0.8 (-0.4 + x),
 0.36 + 1.2 (-0.6 + x), 0.64 + 1.6 (-0.8 + x), 1. + 2. (-1. + x), 1.44 + 2.4 (-1.2 + x),
 1.96 + 2.8 (-1.4 + x), 2.56 + 3.2 (-1.6 + x), 3.24 + 3.6 (-1.8 + x), 4. + 4. (-2. + x)}
```

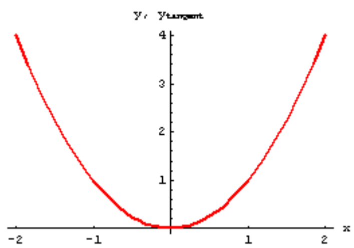
In[224]:=

```
p1 = Plot[Evaluate[listoftangentlines],
  {x, x0, xf},
  PlotStyle → {RGBColor[0, 0, 1]},
  AxesLabel → {"x", "y, ytangent"}];
```



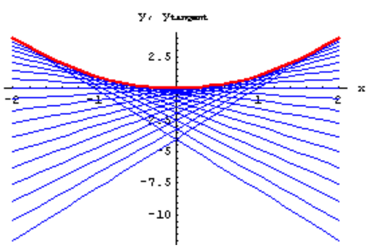
In[225]:=

```
p2 = Plot[y, {x, x0, xf}, PlotRange → All,
  PlotStyle → {RGBColor[1, 0, 0],
    Thickness[0.010]},
  AxesLabel → {"x", "y, ytangent"}];
```



In[226]:=

```
Show[{p1, p2}];
```



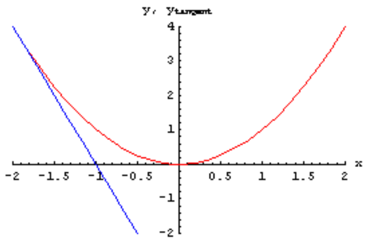
Part V: Making Movies

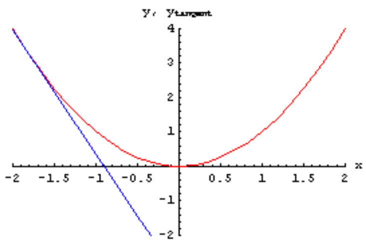
Chapter 3, Section 1

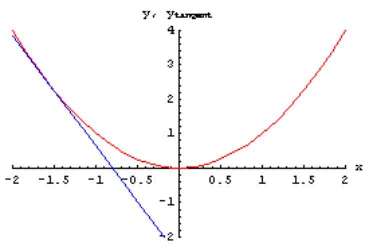
The following command generates a sequence of graphs that you can animate. To animate the graphs, simply place the cursor on any one of the graphs in the sequence and double click the mouse. Or, you can collapse all the cells for the animation sequence into one cell by double clicking the mouse on the first cell bracket that contains all the graphics cells in the sequence, and then type Ctrl+Y. Set the **PlotRange** so that all the graphs in the sequence are the same size; Otherwise, the picture will jump around in the animation. You will have to adjust the vertical limits (in red letters) in the **PlotRange** option to make the animation fit correctly in the viewing window.

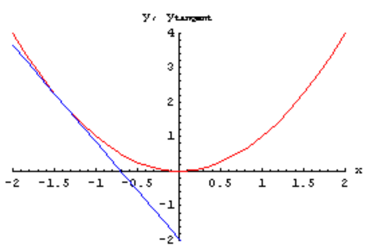
In[227]:=

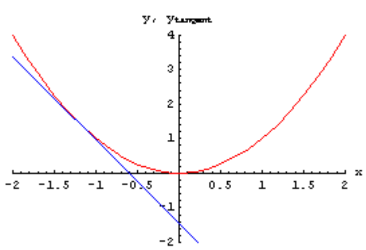
```
Do[Plot[{f[x], tanline[x, a]}, {x, x0, xf},
  PlotRange → {{x0, xf}, {-2, 4}},
  PlotStyle → {{RGBColor[1, 0, 0]},
    {RGBColor[0, 0, 1]}},
  AxesLabel → {"x", "y, ytangent"}],
 {a, x0, xf, 0.2}]
```

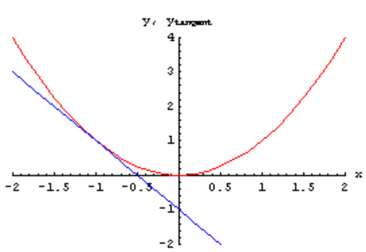

y, ytangent
x

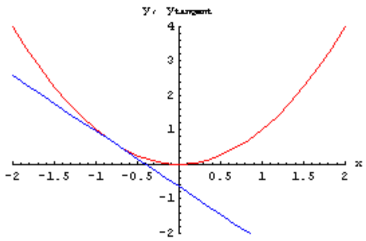

y, ytangent
x

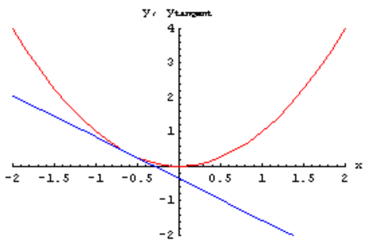

y, ytangent
x

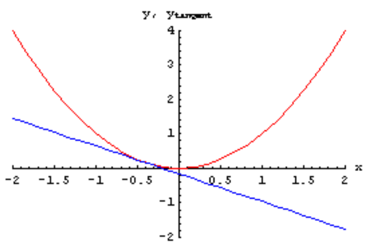

y, ytangent
x

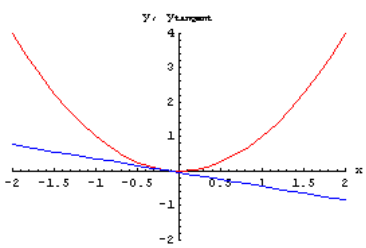

y, ytangent
x

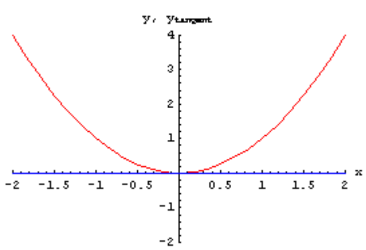

y, ytangent
x

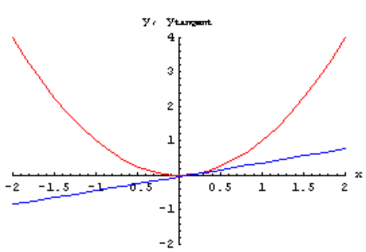

y, ytangent
x
-2 -1.5 -1 -0.5 0.5 1 1.5 2
4 3 2 1 -1 -2


y, ytangent
x
-2 -1.5 -1 -0.5 0.5 1 1.5 2
4 3 2 1 -1 -2


y, ytangent
x
-2 -1.5 -1 -0.5 0.5 1 1.5 2
4 3 2 1 -1 -2


y, ytangent
x
-2 -1.5 -1 -0.5 0.5 1 1.5 2
4 3 2 1 -1 -2


y, ytangent
x
-2 -1.5 -1 -0.5 0.5 1 1.5 2
4 3 2 1 -1 -2


y, ytangent
x
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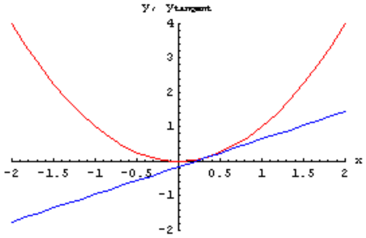
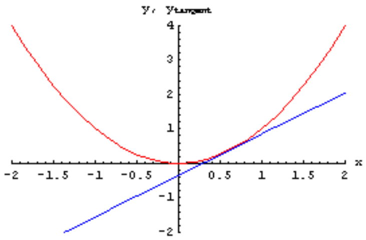
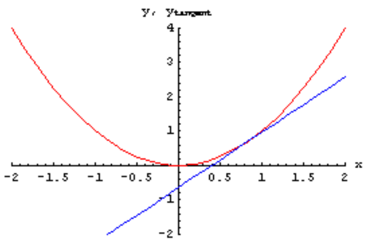
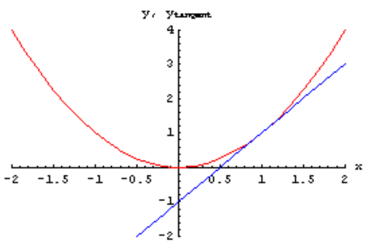
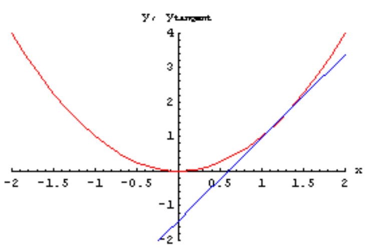
4 3 2 1 -1 -2

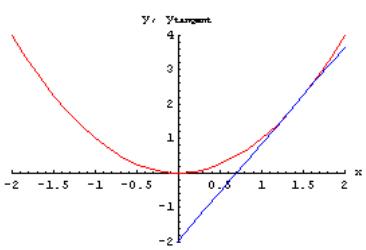

y, ytangent
x

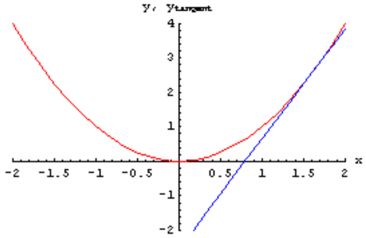

y, ytangent
x

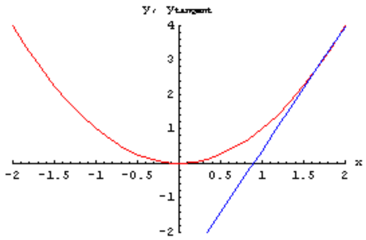

y, ytangent
x

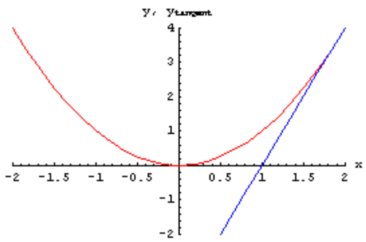

y, ytangent
x


y, ytangent
x


y, ytangent
x







You Try It

First, work through Parts I - V with the example function $f(x)=x^2$, and then repeat the steps in Parts I - V for some functions that you select. Here are some suggestions.

1. x^3 for $-2 \le x \le 2$

2. sin x for $0 \le x \le 2\pi$

3. e^{-x^2} for $-2 \le x \le 2$

4. $\sqrt{x}$ for $0 < x \le 4$ Note that we do not include x=0. Do you know why?