4.7 Dividing Polynomials

Parts of a Division Problem

- There are three parts to a division problem: the **dividend**, the **divisor**, and the **quotient**. A division problem can be written three different ways:

$$\frac{dividend}{divisor} = quotient$$

$$(dividend) \div (divisor) = quotient$$

$$divisor\overline{)dividend}^{\,quotient}$$

Dividing a Polynomial by a Monomial

- To divide a polynomial by a monomial, divide each term of the polynomial by the monomial:

$$\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$$

$$(c \neq 0)$$

Divide: $(12m^6 + 18m^5 + 30m^4) \div 6m^2$

- Divide each term of the polynomial by the monomial:

$$\frac{12m^6}{6m^2} + \frac{18m^5}{6m^2} + \frac{30m^4}{6m^2}$$

- Simplify:

$$2m^4 + 3m^3 + 5m^2$$

Board Examples

- Example 1: $\dfrac{50m^4 - 30m^3 + 20m}{10m^3}$
- Example 2: $\left(-8p^4 - 6p^3 - 12p^5\right) \div \left(-3p^3\right)$
  - *(Hint: Write in descending order 1st)*
- Example 3: $\dfrac{45x^4y^3 + 30x^2y^2 - 60x^2y}{15x^2y}$

Divide: $\left(5x - 8 + 4x^3 - 4x^2\right) \div (2x - 1)$

- Step 1: Rewrite as a long division problem. Make sure that both polynomials are written in descending order, fill in any missing terms with a zero term.

$$2x - 1\overline{)4x^3 - 4x^2 + 5x - 8}$$

- Step 2:
  a) Take the first term of the dividend and divide by the first term of the divisor: $\frac{4x^3}{2x} = 2x^2$ Place the $2x^2$ above it's like term.

$$\begin{array}{r} 2x^2 \\ 2x - 1\overline{)4x^3 - 4x^2 + 5x - 8} \end{array}$$

- Step 2:
  b) $2x^2$ multiplied by $2x - 1 = 4x^3 - 2x^2$. Place the $4x^3 - 2x^2$ below their like terms.

$$\begin{array}{l} \quad\quad\quad\; 2x^2 \\ 2x - 1\overline{)4x^3 - 4x^2 + 5x - 8} \\ \quad\quad\;\; 4x^3 - 2x^2 \end{array}$$

- Step 2:
  - c) Subtract:

$$\begin{array}{r} 2x^2 \\ 2x-1 \overline{)\,4x^3 - 4x^2 + 5x - 8} \\ \ominus\, 4x^3 \oplus\!-\!2x^2 \\ \hline -2x^2 \end{array}$$

- Step 3: Bring down the next term.

$$\begin{array}{r} 2x^2 \\ 2x-1 \overline{)\,4x^3 - 4x^2 + 5x - 8} \\ \ominus\, 4x^3 \oplus\!-\!2x^2 \\ \hline -2x^2 + 5x \end{array}$$

- Step 4: Repeat steps 2 and 3 until you have brought down the last term.

$$\begin{array}{r} 2x^2 - x + 2 \\ 2x-1 \overline{)\,4x^3 - 4x^2 + 5x - 8} \\ \ominus\, 4x^3 \oplus\!-\!2x^2 \\ \hline -2x^2 + 5x \\ \oplus\!-\!2x^2 \ominus\!+ x \\ \hline 4x - 8 \\ \ominus\, 4x \oplus\!-\! 2 \\ \hline -6 \end{array}$$

- Step 5: State your answer. If there is a remainder, place the remainder over the divisor and add it to the quotient.

$$2x^2 - x + 2 + \frac{-6}{2x-1}$$

or

$$2x^2 - x + 2 - \frac{6}{2x-1}$$

- Step 6: Check. Multiply the divisor by the quotient and add the remainder.

$$(2x-1)(2x^2-x+2)+{}^{-}6$$

$$=4x^3-2x^2+4x-2x^2+x-2-6$$

$$=4x^3-4x^2+5x-8$$

Board Examples

- Example 1:
  $Divide \quad 8x^3-4x^2-14x+15 \quad by \quad 2x+3$
- Example 2: $\dfrac{2x^3+5x+x^2+13}{2x+3}$
- Example 3: $Divide \quad x^3-8 \quad by \quad x-2$

Board Examples (continued)

- Example 4: $\dfrac{2m^5+m^4+6m^3-3m^2-18}{m^2+3}$
- Example 5:
  $Divide \quad 3x^3+7x^2+7x+11 \quad by \quad 3x+6$
- Example 6: $(y^3-1)\div(y-1)$

Dividing a Polynomial by a Polynomial

To divide a polynomial by a polynomial, follow the six steps outlined below.

- **Step 1**: Rewrite as a long division problem. Make sure that both polynomials are written in descending order, filling in any missing terms with a zero term.
- **Step 2**:
  a) Divide the first term of the dividend by the first term of the divisor. Place that quotient above it's like term.
  b) Multiply the quotient from part a by the divisor. Place that product below it's like term.
  c) Subtract.
- **Step 3**: Bring down the next term.
- **Step 4**: Repeat steps 2 and 3 until you have brought down the last term.
- **Step 5**: State your answer. If there is a remainder, place the remainder over the divisor and add it to the quotient.
- **Step 6**: Check. Multiply the divisor by the quotient and add the remainder.