

Object: To measure the electrostatic force between two parallel plates and thereby determine the electric permittivity constant (the permittivity of free space  $\epsilon_o$ .)

Theory: Consider two parallel metal plates of area  $A$  and separation  $d$ . If these plates carry equal and opposite charges of magnitude  $q$ , then it can be shown that the electric field between the plates is

$$E_{\text{tot}} = \frac{q}{A\epsilon_o}. \quad (1)$$

When an electric charge is brought into an electric field produced by another charge it experiences a force given by

$$F = qE_{\text{one}}. \quad (2)$$

Thus, the force of attraction between two parallel plates is the charge on one plate multiplied by the field due to the other plate (which is one-half the value given by the above formula). The force attracting the plates together can be found by solving for  $q$  in equation 1 and substituting in equation 2.

$$F = \frac{E^2 A \epsilon_o}{2}. \quad (3)$$

The electric field  $E$  can be calculated from the potential difference  $\Delta V$ :

$$E = \frac{\Delta V}{d}. \quad (4)$$

The force attracting the plates together can thus be written as

$$F = \frac{A(\Delta V)^2 \epsilon_o}{2d^2}. \quad (5)$$

All quantities in this expression except  $\epsilon_o$  are susceptible to measurement. Thus  $\epsilon_o$  is determined.

There are two ways to obtain the plate separation  $d$ : direct measurement with a ruler, and using the formula

$$d = \frac{Da}{2b} \quad (6)$$

where  $D$  is the distance between the laser beam spots for the contact and equilibrium positions of the plates,  $a$  is the perpendicular distance from the line of the knife edges to the center of the plates, and  $b$  is the distance from the mirror to the laser beam spot on the wall.

The value of  $d$  should be small; if it becomes large relative to the area of the plates then edge effects become significant and lack of parallelism increases (and then equation 5 isn't valid).

Apparatus: We'll use a Coulomb balance which has two aluminum plates, one of which balances on a knife edge. Draw a diagram of the apparatus and label appropriate quantities.

Procedure:

1. Measure the area of the plates  $A$ , and the values of  $a$  and  $b$ . Determine the exact center of the movable plate and gently, nondestructively mark it.
2. Mount the plates in the Coulomb balance and take great care to level the apparatus and align the plates (without bending them) so they are parallel.
3. Wire a high-voltage power supply with the positive side to one plate and the negative side to the other plate, but so that no current will flow. Also, in case the two plates accidentally do touch when the power supply is on, place a megohm resistor in one branch of the circuit to limit the current to prevent possible damage to the power supply. Do not yet turn it on.
4. Place a coin on the upper (movable) plate to bring it into contact with the lower plate (this is OK when the power supply isn't on). Note the location of the laser beam on the wall; this is the contact position. Remove the coin.
5. Place a milligram weight (50 mg is a good first weight) on the very center of the movable plate and note the position of the laser beam spot; this is the equilibrium position. Use equation 6 to compute  $d$  and verify with a ruler. Remove the weight.
6. Turn on the high-voltage power supply and gradually increase the voltage to bring the plates back to the equilibrium separation distance. Be careful as this is an unstable equilibrium (the plates will tend to rush together), and you don't want the charged plates to touch each other—you should take great care to insure that they don't. Now the electrostatic force should be equivalent to the milligram weight used to balance the plates. Record your data and turn off the high voltage.
7. Repeat procedures 5–6 for various amounts of weight and corresponding voltage settings, and make a data table containing columns for  $F$  (the weight),  $d$ , and  $\Delta V$ .

Analysis: Plot  $F$  vs.  $(\Delta V)^2/d^2$  and find the least squares slope. Use equation 5 to solve for  $\epsilon_0$ . Compare with the book value,  $8.854187817 \times 10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2$  (which is exact).

Conclusions: If you were a grad student at a rich university and had a whole semester to do nothing but this experiment how would you reduce errors? Go into some detail.