

Object: Determine the charge to mass ratio of the electron by observing its motion in a magnetic field.

Theory: In experimental work involving laboratory-produced magnetic fields, it has been found convenient to use an arrangement of windings known as a Helmholtz coil to generate the needed magnetic field. The Helmholtz set up consists of two coils of equal radius  $R$  placed parallel to each other and separated by a distance equal to the radius of the coils. The magnetic field produced at the center of such a pair of coils is very uniform over a significant region of space in which an experiment can be performed (both  $dB/dx$  and  $d^2B/dx^2$  vanish at the center). The magnitude of this field can be derived from the Biot-Savart Law and is given by

$$B = \frac{8\mu_o NI}{\sqrt{125}R} \quad (1)$$

where  $N$  is the number of turns per coil, and  $I$  is the current in the coils.

If electrons of mass  $m$  and charge  $e$  are accelerated through a potential difference  $V$  and injected into a magnetic field  $\mathbf{B}$  at right angles to it, they will move in a circular orbit having radius

$$r = \frac{\sqrt{\frac{2mV}{e}}}{B}. \quad (2)$$

Equation 2 can be solved for the charge to mass ratio:

$$\frac{e}{m} = \frac{2V}{r^2 B^2}. \quad (3)$$

The electrons will be “boiled off” a heated filament (cathode) in a vacuum tube (see photograph on page 908 in Serway) and then accelerated by an anode plate (at a higher potential) which has a small hole in it. The cathode “gun” is located a small distance  $d$  below the anode plate opening. This causes the center of curvature of the emerging beam to lie just below the anode plate. When conditions are adjusted so that the beam strikes a circle on the anode disk, the true diameter of the beam circle is larger than the recorded value. This results in a recorded value of  $e/m$  that is larger than the actual value. To account for the displacement of the cathode below the anode plate, replace  $r^2$  with  $r'^2$  where  $r'^2 = r^2 + d^2$  and  $d$  is 0.254 cm in our case.

If the electron beam is not exactly perpendicular to the magnetic field the electrons will follow a helical trajectory rather than a circular one. (This is not what you want in this experiment.)

Apparatus: Carefully draw a detailed diagram of the apparatus. Include the wiring.

### Procedure:

1. Derive equations 1 and 2.
2. Determine the radius and the number of windings in each loop of the pair of Helmholtz coils. This data, along with the current, will be used to calculate the magnetic field at the center of the coil arrangement.
3. While the effect of the Earth's magnetic field is not large compared to the Helmholtz field, you may want to consider it and make a correction for it. You should also insure that there aren't other stray magnetic fields nearby (use a compass or gaussmeter).
4. First set the electron tube filament current to be 0.6–0.8 A at 5.0–7.0 V (usually 6.3 V). After a 1-min filament warm-up, the plate potential (accelerating voltage) may be turned on (to about 100 V). You should see a beam of electrons going straight up. Reduce the filament current to the minimum level that still yields a visible beam (higher settings can prematurely burn out the tube which is very costly). Use the grid voltage to focus the beam.
5. Adjust the current in the coils (0–4 A) and the accelerating voltage (80–250 V) so as to produce an electron beam which curves into a visible orbit inside the apparatus tube. Measure the radius of the orbit for each coil current and accelerating voltage setting. The fluorescent circles painted on the anode plate have radii 0.50 cm, 1.0 cm, 1.5 cm, and 2.0 cm. (These radii are the diameters of the electron orbits, taking  $d$  into account.)

### Results:

1. Substitute equation 1 into equation 3 (modified for  $r'^2$ ) and then solve for  $V$  as a function of  $r'^2 I^2$ .
2. A plot of  $V$  vs.  $(r'I)^2$  should produce a straight line whose slope involves  $e/m$ .
3. Determine the least squares slope of your line and solve it for  $e/m$ . Then compare your result with the accepted value of  $e/m$  ( $1.75882 \times 10^{11}$  C/kg), and discuss possible sources of uncertainty.

### Questions:

1. The Millikan oil drop experiment gives  $e$  (current best value is  $1.602176462 \times 10^{-19}$  C), so now compute the mass of the electron.
2. Compute the speed  $v$  of the electrons in this experiment and then determine if a relativistic correction to the mass is needed. Special relativity says  $m = m_o/\sqrt{1 - v^2/c^2}$ , where  $m_o$  is the rest mass.
3. What would be the mass of an electron traveling at half the speed of light? At 99.5% of  $c$ ?
4. What potential difference would an electron have to fall from rest through to acquire a speed of  $3.00 \times 10^6$  m/s (*i.e.*,  $0.10c$ )?
5. How could you determine the size of the error contributed by the earth's magnetic field in this experiment? How could you account for it or counter-act it?