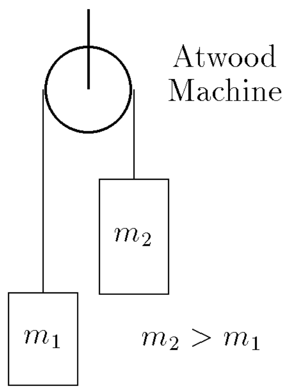
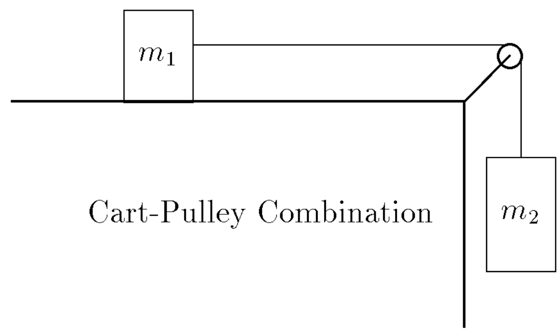
PHSX 221L Accelerated Motion With Two Bodies Name:______________________

Object: To measure the acceleration of an Atwood Machine and that of a cart-pulley mass combination and compare the results with theory.

Theory:

The acceleration of a system can be calculated from Newton's 2nd law, $\boldsymbol{F}_{\text{net}} = m\boldsymbol{a}$, if all the forces are known. In this lab free body diagrams on each block must be constructed and Newton's 2nd law applied to each.





Atwood Machine

The Atwood Machine, named after British mathematician George Atwood, (see example 5.9 in Serway) consists of two unequal masses (say $m_2 > m_1$), one on each side of a suspended pulley and connected by a massless string. If we also assume the friction in the pulley and the mass of the pulley are negligible, then the tension is the same on both sides of the pulley, and the free body diagrams and equations are simple:

$$T - m_1 g = m_1 a, \tag{1}$$

$$m_2 g - T = m_2 a \tag{2}$$

where the positive direction is chosen to be in the direction of the acceleration for each mass, and since they are connected by the string, the magnitudes of the accelerations are equal. Eliminating T from these equations gives the simplified theoretical acceleration of the system as

$$a = \frac{(m_2 - m_1)g}{m_1 + m_2}. \tag{3}$$

If, in successive trials, mass is transferred incrementally from m_1 to m_2 then the numerator will change and the denominator won't. The actual acceleration can be measured (computed) in each case by timing the fall of m_2 from rest a certain distance and using $y = \frac{1}{2}at^2$. Rearranging equation 3 to look a little more like Newton's Second Law as given above produces $(m_2 - m_1)g = (m_1 + m_2)a$

so plotting a graph of $(m_2 - m_1)g$ (*i.e.* the net force) vs. a should give a straight line with a slope of $m_1 + m_2$ (*i.e.* the total mass).

However, if the mass (inertia) of the pulley and friction are not neglected, the tension is not the same on both sides of the pulley and some other terms enter the equations. Equation 1 becomes

$$T_1 - m_1 g = m_1 a. \tag{4}$$

and equation 2 becomes

$$m_2 g - T_2 = m_2 a, \tag{5}$$

There is also now a third equation (good thing, too, since there are more unknowns to solve for) for the pulley. The forces on the pulley will include T_1 and T_2 and F_f (the force of friction of the bearing on the pulley). The right side of Newton's Second Law is trickier, since the pulley isn't accelerating in a translational way, but rather in a rotational way—this means the inertia of the pulley is more complex than its simple mass. The equivalent mass (inertia) of the pulley, m_{eq}, is equal to 1/2 of the pulley's mass if the pulley is a solid disk, *i.e.*, $m_{eq} = \frac{1}{2} m_{pulley}$. Newton's Second Law for the pulley is

$$T_2 - T_1 - F_f = m_{eq} a. \tag{6}$$

Eliminating T_1 and T_2 from equations 4, 5, and 6 gives

$$a = \frac{(m_2 - m_1)g - F_f}{m_1 + m_2 + m_{eq}}. \tag{7}$$

Now a plot of F_{net} for the system (still $(m_2 - m_1)g$) vs. a will produce a straight line with a slope of $m_2 + m_1 + m_{eq}$ and an intercept of F_f. See also example 10.13 in Serway.

Cart-Pulley Combination

This apparatus consists of a cart, which is free to roll or slide on a horizontal plane, connected by a massless string over a pulley to a hanging mass.

Before doing the experimental procedure, do mathematical derivations similar to the ones for the Atwood machine—once assuming a massless, frictionless pulley and a frictionless plane, and then again taking friction and the mass of the pulley into account. (See example 5.14 in Serway.)

Procedure:

Atwood Machine

1. Set up the Atwood apparatus with the figure as your guide. Put the same amount of mass (about 150 g, including the hanger) on each side of the pulley, but use some small masses as part of m_1 so they can be transferred. Use a pan balance to check that m_1 and m_2 are equal to within a few hundredths of a gram. If they aren't, adjust the lighter one with small pieces of paper or tape until they are the same.

2. Estimate the friction in the pulley by adding a small mass, m_f (the exact amount to be determined by trial and error), to m_2 to unbalance the system and cause m_2 to descend at a constant rate. Then $F_f = m_f g$. Remove m_f for the rest of the procedure.

3. Transfer one or two grams of mass from m_1 to m_2. Then, holding m_1 on the cushion, simultaneously release m_1 and start the timer. Stop the timer when m_2 hits the cushion; also immediately stop the pulley by hand. Repeat this measurement a few times. Calculate a from $y = \frac{1}{2}at^2$.

4. Repeat procedure 3 for various larger mass transfers.

Cart-Pulley Combination

1. Set up the horizontal track and adjust it to be level. Pile some weights (masses) on the cart.

2. Estimate the friction in the pulley by putting a small mass, m_f (the exact amount to be determined by trial and error), in place of m_2 which descends at a constant rate. Then $F_f = m_f g$. Remove m_f for the rest of the procedure.

3. Shift mass from the rolling cart to the hanger in increments and measure the acceleration for each situation using $d = \frac{1}{2}at^2$. Keep the total mass (mass of cart plus mass on cart plus hanging mass) constant.

Analysis

For each apparatus (both the Atwood Machine and the Cart-Pulley Combination):

1. Plot the net force vs. acceleration.

2. Use a least squares analysis to compute the slope and the intercept of the line and interpret what these mean physically.

3. Compare the least squares slope with the theoretical value.

Questions:

1. Instead of verifying Newton's Second Law, how could this experiment be used to measure g (as Atwood did)? How accurate would it be?

2. Is there a way to make the Atwood Machine accelerate at a higher rate than 9.8 $\mathrm{m/s^2}$?

3. The Atwood Machine experiment can be reconfigured to keep the force ($(m_2 - m_1)g$) constant while varying the accelerated mass (m_1+m_2) by adding the same amount of mass to both sides. What kind of analysis would be done in this case? What would be the abscissa ($x-$coordinate) and ordinate ($y-$coordinate) and slope of the graph?

4. Using $y = \frac{1}{2}at^2$ assumes that a is constant. Is it? Is it a reasonable approximation? How can you know?

5. How important was it to account for friction and the mass of the pulley?

Conclusions:

Summarize and evaluate your experiment. What did you learn? Think hard.