For all collisions the momentum is conserved (assuming both colliding objects are in the system and there is no net external force on the system). Here we consider the collision between object A and object B. The primed quantities refer to after the collision, unprimed before.

A head-on collision is one-dimensional and allows us to drop the vectors.

$$m_A v_A + m_B v_B = m_A v'_A + m_B v'_B \tag{1}$$

For perfectly elastic collisions the kinetic energy is also conserved.

$$\frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2 = \frac{1}{2} m_A {v'}_A^2 + \frac{1}{2} m_B {v'}_B^2 \tag{2}$$

Multiplying equation 2 by 2 gives

$$m_A v_A^2 + m_B v_B^2 = m_A {v'}_A^2 + m_B {v'}_B^2 \tag{3}$$

We are told that $m_A = m_B$ so we drop subscripts on the m's; in fact, now the m's all cancel. Now equations 1 and 3 become

$$v_A + v_B = v'_A + v'_B \tag{4}$$

and

$$v_A^2 + v_B^2 = {v'}_A^2 + {v'}_B^2 \tag{5}$$

We are also told that $v_B = 0$, which leaves equations 4 and 5 as

$$v_A = v'_A + v'_B \tag{6}$$

and

$$v_A^2 = {v'}_A^2 + {v'}_B^2 \tag{7}$$

Solving this system of two equations for the unknowns v'_A and v'_B gives

$$v'_A = 0 \tag{8}$$

and

$$v'_B = v_A \tag{9}$$

which is as expected. And $\epsilon = 1$.

This is derived in more general terms in *University Physics* by Crummet and Western, page 236. They derive some "working equations" in terms of ϵ and $\mathcal{M}$ where $\mathcal{M} = m_B/m_A$.

$$v_A + \mathcal{M}v_B = v'_A + \mathcal{M}v'_B \tag{10}$$

$$\epsilon(v_A - v_B) = (v'_B - v'_A) \tag{11}$$

Students are asked in problem 36 on page 245 to solve for the final velocities, yielding

$$v'_A = \frac{1 - \mathcal{M}\epsilon}{1 + \mathcal{M}}v_A + \frac{\mathcal{M}(1 + \epsilon)}{1 + \mathcal{M}}v_B \tag{12}$$

$$v'_B = \frac{1 + \epsilon}{1 + \mathcal{M}}v_A + \frac{\mathcal{M} - \epsilon}{1 + \mathcal{M}}v_B \tag{13}$$

It is easy to use equations 12 and 13 to verify results 8 and 9 for a head-on perfectly elastic collision ($\epsilon = 1$) between objects of equal mass ($\mathcal{M} = 1$) with $v_B = 0$.

Now suppose $\epsilon = 0.8$ (and everything else is as in the previous example).

$$v'_A = \frac{1 - 0.8}{1 + 1}v_A = \frac{0.2}{2}v_A = 0.1v_A \tag{14}$$

$$v'_B = \frac{1 + 0.8}{1 + 1}v_A = \frac{1.8}{2}v_A = 0.9v_A \tag{15}$$