Concept Question 8
Figure 9.12(a) shows the momentum vectors of two particles A and B before they collide. Figure 9.12(b) shows the momentum vector for particle A after the collision. Sketch the momentum vector for particle B after the collision assuming momentum is conserved.

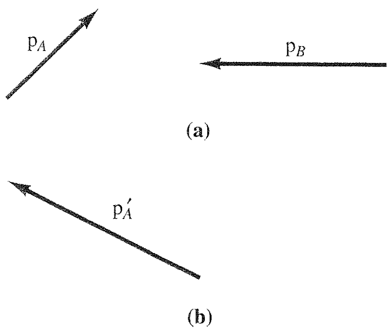

FIGURE 9.12

(a) The momentum vectors of particles A and B before the particles collide.
(b) The momentum vector of particle A after the collision. Can you draw the momentum vector for particle B following the collision?

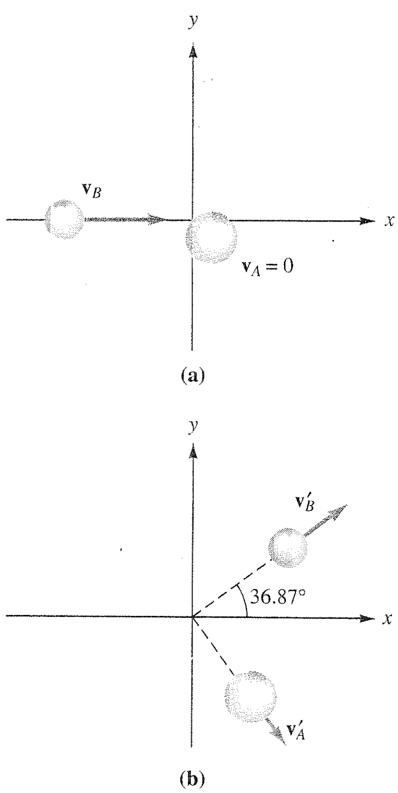

FIGURE 9.13

Two unequal masses collide while sliding over a frictionless surface. Vector momentum is conserved, but kinetic energy may not be.

Perfectly Elastic Collisions

In the case of perfectly elastic collisions kinetic energy is conserved. This condition provides a third equation to be satisfied by the colliding particles. Given the initial speeds and directions of the two approaching particles, three equations are insufficient to solve for the final speeds and directions of both particles after collision because there are four unknowns after the collision: two speeds and two directions.

It makes physical sense that we cannot determine the result of a two-dimensional collision from the initial momenta only, because the results of such a collision must depend not only on the original velocities, but also on the location on the objects where they come into contact. If you have played pool or shuffleboard, you know that the result of a two-dimensional collision depends strongly on the point of contact of the objects. Two-dimensional collisions can range anywhere from head on to a barely glancing impact. In addition, the details of how the collision force depends on the separation of the particles during the collision influences the result.

The realm of atomic physics is one in which perfectly elastic collisions do occur. The molecules of a gas may collide in perfectly elastic collisions. (Such collisions may also be partially elastic if the molecules are excited to higher internal energy levels. See Chapter 21 for more details.) If sufficient information can be obtained concerning the direction of particles after a collision, their momenta can be deduced.

EXAMPLE 9.16 *Now for Some Air Hockey*

On a frictionless air-hockey table two pucks collide in what is, for all practical purposes, a perfectly elastic collision. Puck A, which has a mass twice that of puck B, is initially at rest. Before the collision, puck B moves with an initial velocity of $v_A =$ 5.00 m/s along the x-axis. After the pucks collide, puck B is observed to travel at an angle $\theta = 36.87°$ above the positive x-axis. Find the speeds of both pucks and the direction of travel for puck A after the collision.

SOLUTION The masses are unknown, so we let one puck have mass m and the other mass $2m$. Because no net external force acts in the horizontal plane, momentum is conserved in this plane. Therefore, we expect to obtain two equations, one from equating the total x-momentum before the collision to the total x-momentum after the collision, and a second from equating the total y-momentum before the collision to the total y-momentum after the collision.

Momentum before = Momentum after

x-momentum: $$mv_A = mv'_B\cos(\theta) + 2mv'_{A_x}$$

y-momentum: $$0 = mv'_B\sin(\theta) + 2mv'_{A_y}$$

Because we are modeling the collision as if it were perfectly elastic, we may also equate the kinetic energy before and after the collision:

kinetic energy: $$\frac{1}{2}mv_A^2 = \frac{1}{2}mv_B'^2 + \frac{1}{2}(2m)v_A'^2$$

We may simplify the above equations by dividing each by m. The kinetic energy equation can be multiplied by 2. We also note that $v_A'^2 = v_{A_x}'^2 + v_{A_y}'^2$. The resulting three equations have three unknowns:

$$v_A = \cos(\theta)\,v'_B + 2v'_{A_x} \qquad (9.13)$$

$$0 = \sin(\theta)\,v'_B + 2v'_{A_y} \qquad (9.14)$$

$$v_A^2 = v_B'^2 + 2v_{A_x}'^2 + 2v_{A_y}'^2 \qquad (9.15)$$

We solve the first two equations in this last set for v'_{A_x} and v'_{A_y}, respectively, then substitute the results into the third equation to obtain

$$v_A^2 = v_B'^2 + (v_A - \cos(\theta)\, v_B')^2 + (\sin(\theta)\, v_B')^2$$

When numerical values are substituted, the above equation reduces to

$$(3.00)v_B'^2 - (8.00 \text{ m/s})v_B' - (25.00 \text{ m}^2/\text{s}^2) = 0,$$

which we solve using the quadratic formula to obtain

$$v_B' = \begin{cases} +4.51 \\ -1.85 \end{cases} \text{m/s}$$

Because v_B' is the magnitude of the vector $\mathbf{v}_B'$, it must be a positive quantity; thus, we choose the positive root. Substituting $v_B' = 4.51$ m/s into Equations (9.13) and (9.14), we obtain $v_{A_x}' = 0.695$ m/s and $v_{A_y}' = -1.35$ m/s. Prudence dictates that we should substitute these three values into Equation (9.15) to check our solution: $25.00 \stackrel{?}{=} (4.51)^2 + 2(0.695)^2 + 2(-1.35)^2 = 24.95$, which checks well enough.

Finally we can find the angle made by puck A and the x-axis from

$$\phi = \arctan\left(\frac{v_{A_y}'}{v_{A_x}'}\right) = \arctan\left(\frac{-1.35}{0.695}\right) = -62.8°$$

Concept Question 9
If you can hear the collision between two objects, can the collision be perfectly elastic? Explain.

Partially Elastic Collisions

In the case of partially elastic collisions, kinetic energy is not conserved. Therefore, we have only the conservation of momentum condition to apply to our problem. In two dimensions this means two equations. Thus, we can solve problems with no more than two unknowns. Typically you might know everything about all the participants in the collision except the final velocity of one. You could then solve for the magnitude and direction of the velocity for that one object. We do not show a detailed example for this case. Look at the first steps in Examples 9.15 and 9.16 and you will see how to write the conservation of momentum condition for a collision in two dimensions.

9.7 Summary

Impulse is defined as integral of force with respect to time.

$$\mathbf{J} = \int_{t_o}^{t_f} \mathbf{F}\, dt \tag{9.1}$$

Momentum is mass times velocity, $\mathbf{p} = m\mathbf{v}$. The **impulse–momentum theorem** states that the net impulse on an object with a constant mass is equal to the object's change in momentum.

$$\mathbf{J}_{\text{net}} = \Delta\mathbf{p} \tag{9.2}$$

Newton's second law can be written in terms of the time rate of change of momentum

$$\mathbf{F}_{\text{net}} = \frac{d\mathbf{p}}{dt} \tag{9.4}$$

The relation between the **relative velocities** of three objects can be written

$$\mathbf{v}_{AC} = \mathbf{v}_{AB} + \mathbf{v}_{BC} \tag{9.6}$$

The **conservation of momentum** law states that when the net external force on a system is zero, the total momentum of the system is constant. When this condition is