**PHSX 221L** **Measuring Velocity** **Name:**_____

Object: To determine the velocity of a projectile by two independent methods: (1) measuring a trajectory, and (2) using a ballistic pendulum; and then to compare the results.

Theory: When a projectile (of mass $m_1$) is fired into a ballistic pendulum (of mass $m_2$) the impact causes the pendulum and embedded projectile (total mass $m_1 + m_2$) to swing up to a maximum height. Kinetic energy is not conserved in the inelastic collision, but momentum is. And after the collision mechanical energy is conserved as the kinetic energy of the pendulum immediately after the impact is changed completely into potential energy at the maximum height. These conservation laws can be used to derive an expression for the initial velocity $v_{1i}$ of the projectile.

Conservation of momentum *during* the collision gives:

$$m_1 v_{1i} = (m_1 + m_2) v_f \tag{1}$$

where $m_1$ is the mass of the ball, $v_o$ is the muzzle velocity of the ball, $m_2$ is the mass of the catcher, and $v$ is the velocity of the catcher (and ball) after collision.

Conservation of mechanical energy *after* the collision gives:

$$\frac{1}{2}(m_1 + m_2) v_f^2 = (m_1 + m_2) g h \tag{2}$$

where $h$ is height above the starting point.

Since the collision is inelastic, only a fraction $F$ of the initial kinetic energy of the projectile is transferred to the system after impact.

$$(F)\left(\frac{1}{2}\right) m_1 v_{1i}^2 = \frac{1}{2}(m_1 + m_2) v_f^2 \tag{3}$$

where $v_f$ is the velocity of the combined mass after the collision. If conservation of linear momentum is used (equation 1), it can be shown that

$$F = \frac{m_1}{m_1 + m_2}. \tag{4}$$

Procedure:

1. Solve (2) for $v_f$ and then (1) for $v_{1i}$ and verify by comparison to Example 9.6 in Serway.
2. Using equations of motion in both the $x-$ and $y-$direction, derive an expression for the initial velocity of the projectile in terms of the distance of fall and the horizontal range (and $g$) by eliminating $t$ between the equations.
3. Fire the projectile into the ballistic pendulum a number of times and calculate the initial projectile velocity from your derived equation.

4. Fire the projectile off onto the floor a number of times and note the horizontal range and the height from which it was fired. Calculate the initial projectile velocity from your derived equation.

Analysis:

1. Compare the two results. (Compute percent difference.)
2. How precise was your data?
3. Note that Equation 3 may be solved for $F$, thus giving the ratio of the two kinetic energies. Compute the kinetic energies before and after impact, and find the fraction $F$ by computing the ratio of the two energies. Compare this value of $F$ with the theoretical value predicted by Equation 4. (Find the percent difference between the two values.)

Conclusions: Summarize and evaluate your experiment. Include mention of likely sources of error and how you dealt with them.