

Object: To measure the rotational inertia of a wheel and axle combination and compare with theory.

Theory: The rotational inertia (or moment of inertia) of an object is defined by the integral:

$$I = \int r^2 dm. \quad (1)$$

The result for a disc is  $I = \frac{1}{2}MR^2$  (around the axis of symmetry), with different formulæ for other geometrical shapes.

Newton's second law of motion for rotating objects can be written as

$$\tau_{\text{net}} = I\alpha. \quad (2)$$

Apparatus: The apparatus for this experiment consists of a heavy metal wheel and cylindrical axle. The wheel is mounted on a frame which allows it to rotate freely about a fixed axis. A string is wrapped around the outside of either the wheel or the axle with a known mass attached and allowed to hang vertically. When the hanging mass is released the tension in the string exerts a torque on the wheel, causing it to rotate with an angular acceleration as the mass falls.

Procedure:

1. As usual, draw a picture of the apparatus. Indicate on the diagram the important dimensions of the system.
2. Safety tip: do not put your fingers near the system when it is rotating or the little nib can hurt your finger.
3. Wrap a string around the outside of the wheel and attach a small mass that will cause the wheel to rotate without acceleration. From this determine the frictional torque in the bearings, and then treat it as you did the frictional force in the Atwood Machine lab. You should get about the same frictional torque wrapping the string around the axle.
4. Attach a larger mass to the string and measure the time required to fall a predetermined distance while accelerating (use something soft to cushion the fall at the bottom). From the time and distance compute the linear acceleration (assumed constant) using  $y = \frac{1}{2}at^2$ . The angular acceleration  $\alpha$  is then simply  $a/r$ . Also compute the tension  $T$  in the string by making a free body diagram for the hanging mass and applying  $F = ma$ ; this tension is the force in the applied torque  $\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$ . See example 10.12 in Serway.
5. Collect more data points by repeating step 4 for different hanging weights and also for the string wrapped around the axle (smaller radius).

### Analysis:

1. The mass of the wheel-axle combination is printed on the wheel; however you cannot use the simple formula  $I = \frac{1}{2}MR^2$  for the compound system at once. Instead, compute the theoretical value of  $I$  for the system by adding the rotational inertias of the various parts (wheel and axle) of the system—each of which is a disk or cylinder to which  $I = \frac{1}{2}MR^2$  can be applied individually. Assume a constant density  $\rho$  so that the ratio of the masses of the wheel and axle is equal to the ratio of the volumes. Don't count the chunk of metal in the very middle of the system twice.
2. From the results of procedures 4 and 5, make a single graph of  $\tau_{\text{net}}$  vs.  $\alpha$  (this should remind you of a graph of  $F$  vs.  $a$  that you made in a previous lab). All of your data should be plotted on the same graph. The least-squares slope is your experimental value of the rotational inertia of the wheel-axle combination; report it to the proper number of significant digits.
3. How do your computed and measured values of  $I$  for the system compare?

### Conclusions:

1. What problems did you encounter in this lab and how did you overcome them?
2. How has this lab helped you understand rotational inertia?
3. Could the rotational inertia for oddly shaped objects be determined using the methods in this lab? How?