

Title: Torques and Equilibrium

Object: To study center of gravity, equilibrium, and the effects of different placement of forces.

Introduction: A torque results from a force \mathbf{F} acting at a distance \mathbf{r} from a specific point; torques produce or tend to produce rotational acceleration about the point. Torque is a vector quantity given by

$$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}; \quad (1)$$

its magnitude is given by $\tau = rF \sin \theta$ where θ is the angle between \mathbf{r} and \mathbf{F} , and its direction by the right hand rule. The angle θ will be 90° for the torques in this experiment, so the math is simplified. The official S.I. unit for torque is N·m, but it would be convenient to use units of g·cm in this experiment.

A body is said to be in equilibrium if there is no net force and no net torque acting on the body. Stated mathematically, a body is in equilibrium if

$$\Sigma \mathbf{F} = 0 \quad (\text{translational equilibrium}) \quad \text{and} \quad \Sigma \boldsymbol{\tau} = 0 \quad (\text{rotational equilibrium}). \quad (2)$$

Rotational equilibrium implies that the magnitude of the sum of the counter clockwise toques is equal to the magnitude of the sum of the clockwise torques: $\Sigma \tau_{CCW} = \Sigma \tau_{CW}$.

The center of gravity of a body is defined as the point at which a single upward force can balance the gravitational attraction on all parts of the body for any orientation of the body. An equivalent definition is the point through which the sum of all the torques due to the gravitational forces on all the little elements (parts) of mass of the body is zero. Effectively, the entire mass of the body can be assumed to act at the center of gravity. If the gravitational field is uniform throughout the body then the center of gravity coincides with the center of mass. A body that is uniform and symmetric has its center of gravity at the center of symmetry.

Procedure:

Center of Gravity of Meter Stick

1. Use a laboratory balance to find the mass of a meter stick.
2. Suspend the meter stick in a knife-edge clamp and carefully slide it back and forth until it balances. Record the corresponding marking on the meter stick as the center of gravity.
3. Estimate the uncertainty of this measurement by moving the stick slightly back and forth in its support to see at what point it deviates from horizontal.
4. Now suspend your meter stick at the 30.0 cm mark. Put a clamp at the 5.0 cm mark and hang enough mass on a hanger there to achieve equilibrium. Record the total amount of mass (including clamp and hanger) at the 5.0 cm mark on your accurate and clear diagram.

5. Compute the mass of the meter stick by assuming the counterclockwise torques equal the clockwise torques when the stick is in equilibrium (remember, the torque due to the weight of the meter stick is calculated as if all of the meter stick's mass were concentrated at its center of gravity).
6. Compute the percent difference between the mass of the meter stick as derived from the equilibrium equations and its mass from the laboratory balance.
7. Fold an eighth-sheet of paper in half a couple of times and place it on the meter stick at the 99 cm mark and note the results.

Equilibrium

1. Suspend the meter stick at the 35.0 cm mark. Using the clamps and hangers, suspend a mass of about 230.0 g (including clamp and hanger) at the 5.0 cm mark, and a mass of about 100.0 g (including clamp and hanger) on the other side of the suspension point. Move this second mass until equilibrium is achieved, and then record the total masses, including the knife-edge clamps, and their positions.
2. Using equation 1 compute the total counterclockwise torque and the total clockwise torque. Remember to include the torque due to the weight of the meter stick itself.
3. Compute the percent difference between the counterclockwise and the clockwise torques.
4. Suspend the meter stick at its center of gravity found in the first part of the procedure. Hang a mass of about 200.0 g (including clamp and hanger) at the 20.0 cm mark and a mass of about 100.0 g (including clamp and hanger) at the 55.0 cm mark. Find a third mass that will put the system in equilibrium when it is on the same side of the support as the 100 g mass.
5. Using equation 1 compute the total counterclockwise torque and the total clockwise torque.
6. Compute the percent difference between the counterclockwise and the clockwise torques.

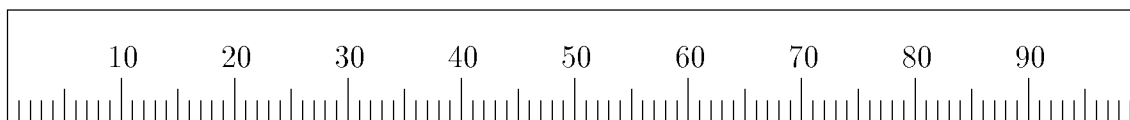
Value of Unknown Mass

1. Suspend the meter stick at its center of gravity found in the first part of the procedure. Hang the unknown mass at the 10.0 cm mark. Then hang a known mass on the other side of the suspension point and slide it back and forth until equilibrium is achieved.
2. Use the principles of equilibrium (the torques must sum to zero) to determine the value of the unknown mass. Take the knife edge clamps into account.
3. Weigh (mass) the unknown mass on the beam balance; note the similarity in operation between your meter stick "balance" and the laboratory balance.
4. Compute the percent error between the two determinations of the mass.
5. Repeat this procedure for your own unknown mass, such as a set of keys or a calculator, if your instructor says there is enough time.

Results:

Center of Gravity of Meter Stick

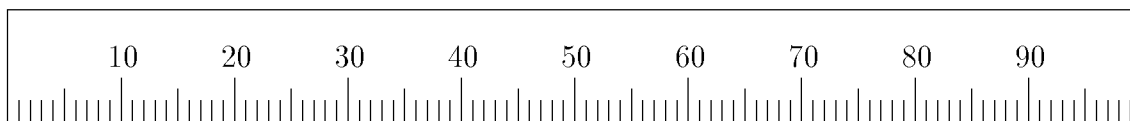
1. Mass of stick from laboratory balance: _____
2. Location of center of gravity of meter stick: _____
3. Estimate of uncertainty: _____
4. Sketch stick suspension point and precise locations and masses of the various weights that create torques.



5. Computed mass of meter stick from equilibrium equations: _____
6. Percent difference between two determinations of mass of meter stick: _____
7. Observations:

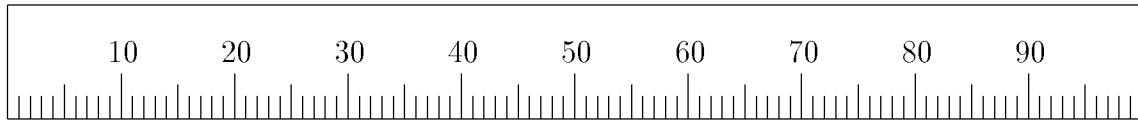
Equilibrium

1. Sketch stick suspension point and precise locations and masses of the various weights that create torques.



2. Torque totals: $\Sigma\tau_{CCW} =$ _____ $\Sigma\tau_{CW} =$ _____
3. Percent difference between CCW and CW torques: _____

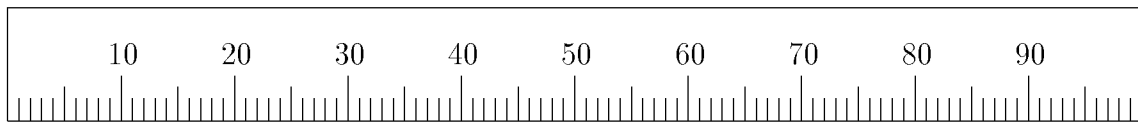
4. Sketch stick suspension point and precise locations and masses of the various weights that create torques.



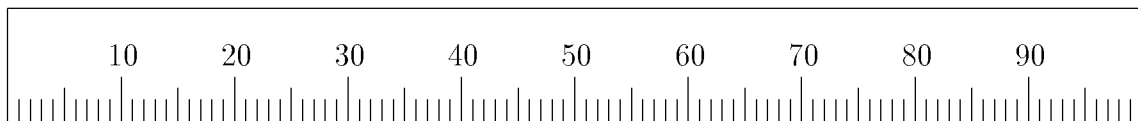
5. Torque totals: $\Sigma\tau_{CCW} =$ _____ $\Sigma\tau_{CW} =$ _____
 6. Percent difference between CCW and CW torques: _____

Value of Unknown Mass

1. Sketch stick suspension point and precise locations and masses of the various weights that create torques.



- | | lab's unknown | your unknown |
|---|---------------|--------------|
| 2. Value of unknown mass from torque calculation: | _____ | _____ |
| 3. Value of unknown mass from lab balance: | _____ | _____ |
| 4. Percent Difference: | _____ | _____ |
5. Sketch stick suspension point and precise locations and masses of the various weights that create torques.



Discussion Questions:

1. If you were careful (both in taking data and analyzing it) during this lab, your percent differences will be very small—less than in most prior experiments, although you used the same equipment, such as meter sticks, balances, etc. Why are the percent differences so low in this experiment?
2. What evidence supports the idea that the weight of an extended object acts at the center of gravity?
3. In a couple of procedures you had to include the torque due to the weight of the meter stick itself; in others you didn't; why?
4. In this experiment you used mass units on force quantities. Usually this would be considered incorrect and would lead to incorrect results when used in computations. Why were you able to get away with this in this experiment?
5. Could you use a modified “meter stick balance” to measure the very small masses (thousandths of a gram)? How?

6. In step 2.1 of the procedure you calculated torques about the 35 cm mark. Recalculate your torques about the 0.0 cm mark. Find all the CW and then all the CCW torques and see if the totals equal. (Don't forget to add the torques exerted at the suspension point: the weight of the meter stick and all the masses on it contributes a CW torque and the normal force exerted upward by the suspension device contributes a CCW torque.) Recalculating torques about a new axis is an easy way to check your equilibrium calculations.

Conclusions: