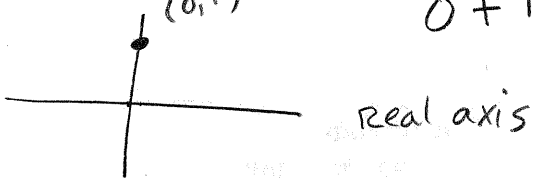
12/5/12 - Sec 6.4

imaginary axis

$(0,1) = i$



$a + bc$

$0 + 1i$

#35

Cube Roots of i

$z = 1(\cos 90 + i \sin 90)$ $\quad r = 1, \quad \theta = 90°$

$$\sqrt[3]{1}\left(\cos\frac{90+360\cdot k}{3} + i\sin\frac{90+360\cdot k}{3}\right)$$

$k=0$ $\quad 1\left(\cos\frac{90+0}{3} + i\sin\frac{90+0}{3}\right)$

$1(\cos 30 + i \sin 30)$ ← 1st

$k=1$ $\quad 1\left(\cos\frac{90+360\cdot 1}{3} + i\sin\frac{90+360}{3}\right)$

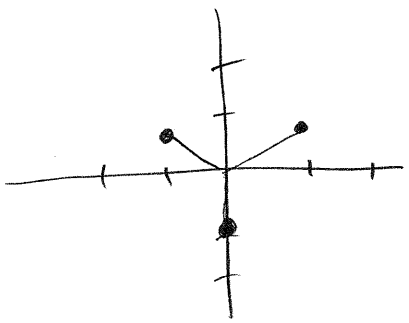
$1(\cos 150 + i \sin 150)$ ← 2nd

$k=2$ $\quad 1\left(\cos\frac{90+360\cdot 2}{3} + i\sin\frac{90+360\cdot 2}{3}\right)$

$1(\cos 270 + i \sin 270)$ ← 3rd

$-i$

$\frac{\sqrt{3}}{2} + \frac{1}{2}i, \quad -\frac{\sqrt{3}}{2} + \frac{1}{2}i, \quad 0 + -1i$



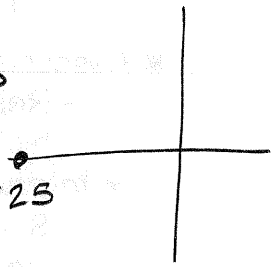
#42 $x^3 + 125 = 0$

$\sqrt[3]{x^3} = \sqrt[3]{-125}$

Looking for 3 $\sqrt[3]{}$ of -125

$a + bi$

$-125 + 0i$



$z = 125(\cos 180 + i \sin 180)$ $r = -125$

$r = 125$

$\theta = 180$

check: $125(-1 + i \cdot 0)$

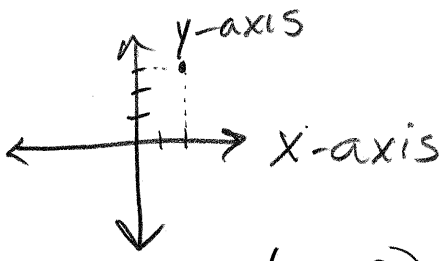
$-125 + 0i$

$$\sqrt[3]{125}\left[\cos\frac{180 + 360 \cdot k}{3} + i\sin\frac{180 + 360 \cdot k}{3}\right]$$

Leave to finish

Sec 6.4 Polar Equations

The Polar Coordinate System



← Rectangular coordinate systems

(2, 3)

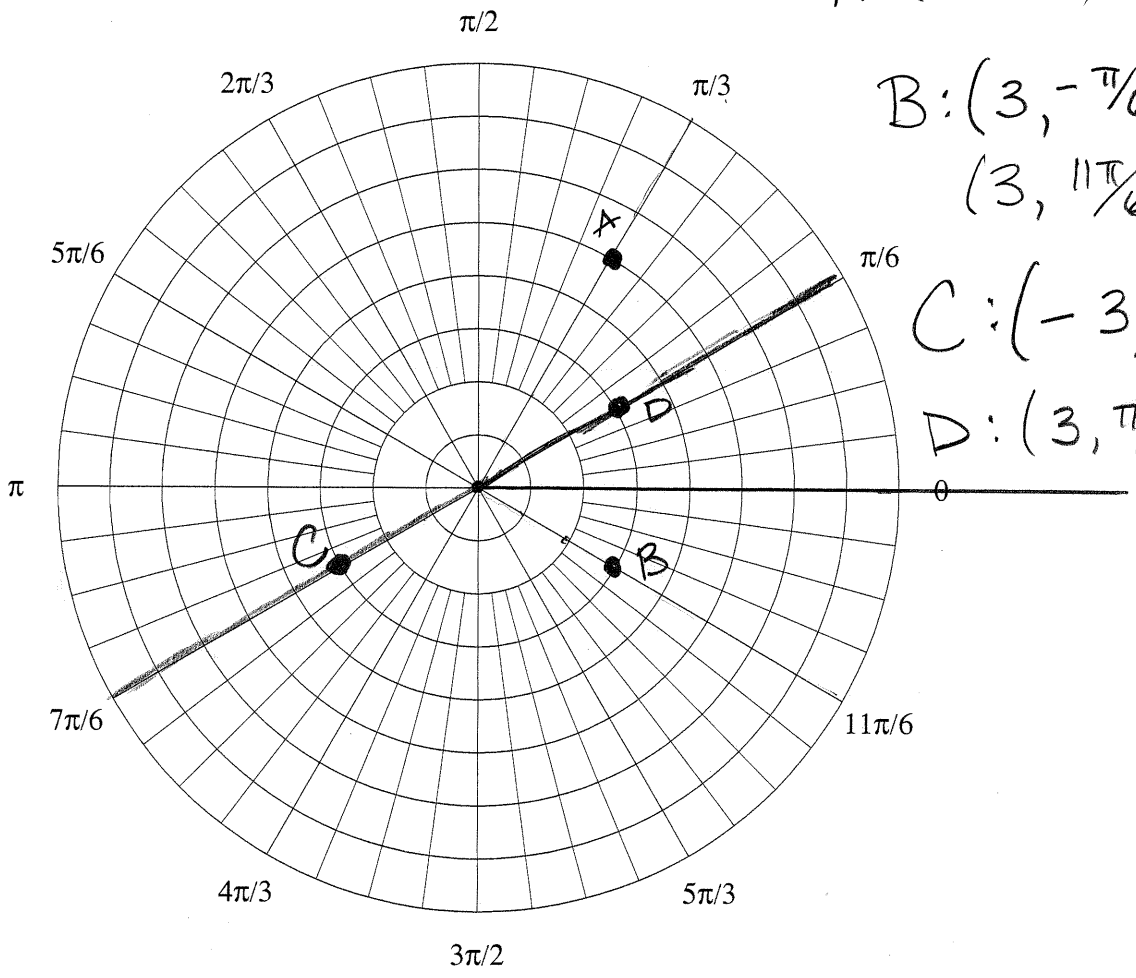
(↑ ↑)

(x, y)

Requires

Pole is our starting point

Polar axis is a line used to start angles from

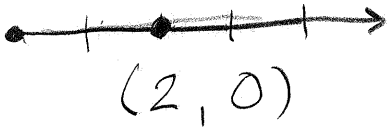


A: $(5, \pi/3)$

B: $(3, -\pi/6)$
$(3, 11\pi/6)$

C: $(-3, \pi/6)$

D: $(3, \pi/6)$



Coordinate in Polar

(r, θ)

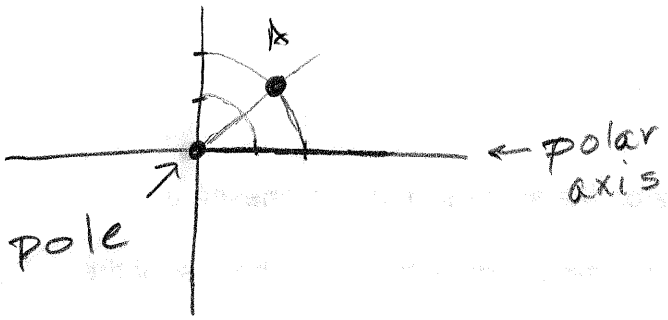
↑ distance to pole

↑ angle in standard position with initial side on polar axis.

See polar Graph paper

No Graph Paper

Lay Polar over Rectangular



$A : (2, \pi/4)$

$\sin\theta = \frac{y}{r}$ ⇓ $y = r\sin\theta$

$\cos\theta = \frac{x}{r}$ ⇓ $x = r\cos\theta$

$\tan\theta = \frac{y}{x}$ ↑ Rect → polar

Rect → polar

$x^2 + y^2 = r^2$

Polar : $(2, \pi/3)$ ($r=2$, θ)

Rect : $(1, \sqrt{3})$

$x = 2\cos\pi/3$
$= 2(1/2) = 1$

$y = 2\sin\pi/3$
$2(\sqrt{3}/2) = \sqrt{3}$

Rect: $(3, 7)$

Polar: $(\sqrt{58}, 1.17)$

$r^2 = x^2 + y^2$

$r^2 = 3^2 + 7^2$

$= 9 + 49$

$r^2 = 58$

$\tan\theta = \frac{y}{x}$

$\tan\theta = \frac{7}{3}$

$\theta = \tan^{-1}\left(\frac{7}{3}\right)$

$= 1.17$

↑ Quad I ✓

Polar Equations:

Equations involving r, θ

$r = 4\sin\theta$

Build table

θ	$\pi/6$	$\pi/4$	$\pi/3$				
r	2	$2\sqrt{2}$	$2\sqrt{3}$				

Spiral of Archemedes

$r = \theta$

Leaf Roses

$r = 3\sin 2\theta$

$= 3\sin 3\theta$

Circle: $x^2 + y^2 = 1$