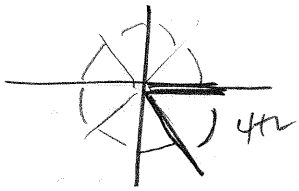
10/12/2012 - Sec 3.5 (cont.) Math 10Ded

#56 $\sin\left(\frac{\alpha}{2}\right) = -\frac{1}{3}$ $\left\{\frac{7\pi}{4} < \frac{\alpha}{2} < 2\pi\right.$

find $\sin\alpha$, $\cos\alpha$, $\tan\alpha$, $\csc\alpha$, $\sec\alpha$, $\cot\alpha$

Want → α

Given → $\frac{\alpha}{2}$



$\frac{\alpha}{2} = \theta$ $\quad \sin\theta = -\frac{1}{3}$

$\alpha = 2\theta$ $\quad \sin 2\theta$

2 2 2

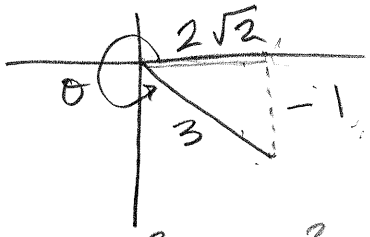
$\frac{7\pi}{4} < \theta < 2\pi$

$\frac{7\pi}{2} < 2\theta < 4\pi$

$\sin 2\theta = 2\sin\theta\cos\theta$

$2\left(-\frac{1}{3}\right)\frac{2\sqrt{2}}{3}$

$\sin 2\theta = -\frac{4\sqrt{2}}{9}$



$x^2 + (-1)^2 = 3^2$

$x^2 + 1 = 9$

$\sqrt{x^2} = \sqrt{8}$

$x = \pm\sqrt{8}$

$2\sqrt{2}$

$\cos\alpha = \cos 2\theta = 1 - 2\sin^2\theta$

$= 1 - 2\left(-\frac{1}{3}\right)^2$

$= 1 - 2\cdot\frac{1}{9}$

$= 1 - 2/9 = 7/9$

$\tan\alpha = \frac{-4\sqrt{2}/9}{\mp/9} = \frac{-4\sqrt{2}}{9}\cdot\frac{9}{7}$

$\sin\alpha = \frac{-4\sqrt{2}}{9}$ $\quad \cos\alpha = \frac{7}{9}$ $\quad \tan\alpha = \frac{-4\sqrt{2}}{7}$

$\csc\alpha = \frac{-9\cdot\sqrt{2}}{4\sqrt{2}\cdot\sqrt{2}} = \frac{-9\sqrt{2}}{8}$ $\quad \sec\alpha = 9/7$ $\quad \cot\alpha = \frac{-7\cdot\sqrt{2}}{4\sqrt{2}\cdot\sqrt{2}} = \frac{-7\sqrt{2}}{8}$

$$\sin\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{2}} \quad \text{or} \quad \sin\theta = \pm\sqrt{\frac{1-\cos 2\theta}{2}}$$

$$\cos\frac{x}{2} = \pm\sqrt{\frac{1+\cos x}{2}} \quad \text{or} \quad \cos\theta = \pm\sqrt{\frac{1+\cos 2\theta}{2}}$$

$$\tan\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{1+\cos x}} \quad \text{or} \quad \tan\theta = \pm\sqrt{\frac{1-\cos 2\theta}{1+\cos 2\theta}}$$

$$= \frac{\sin x}{1+\cos x} \qquad = \frac{\sin 2\theta}{1+\cos 2\theta}$$

$$= \frac{1-\cos x}{\sin x} \checkmark \qquad = \frac{1-\cos 2\theta}{\sin 2\theta}$$

#11 $\tan 15^\circ = \tan\frac{30^\circ}{2} = \frac{1-\cos 30}{\sin 30}$

$= \frac{1-\cos(2\cdot 15)}{\sin(2\cdot 15)}$ → $= \frac{\left(1-\sqrt{3}/2\right)\cdot 2}{1/2 \cdot 2}$

$\frac{2-\sqrt{3}}{1}$

$= 2-\sqrt{3}$

$\cos 30$

$\frac{1}{2}, \frac{\sqrt{2}}{2}, \frac{\sqrt{3}}{2}$

#15 $\sin(22.5^\circ) = \sin\left(\frac{45}{2}\right)$

$$= \pm\sqrt{\frac{1-\cos 45}{2}}$$

$$= \sqrt{\frac{\left(1-\sqrt{2}/2\right)\cdot 2}{(2)\cdot 2}}$$

$$\sqrt{\frac{2-\sqrt{2}}{4}}$$

$$= \frac{\sqrt{2-\sqrt{2}}}{\sqrt{4}}$$

$$= \boxed{\frac{\sqrt{2-\sqrt{2}}}{2}}$$

52 Find all 6 trig functions of α

Given $\cos 2\alpha = \frac{1}{3}$ $\quad 360^\circ < 2\alpha < 450^\circ$

Want → α $\quad \cos\alpha = \pm\sqrt{\frac{1+\cos 2\alpha}{2}}$ $\quad \frac{360}{2} < \frac{2\alpha}{2} < \frac{450}{2}$

Given → 2α

$$= \pm\sqrt{\frac{1+\frac{1}{3}}{2\cdot 3}} \qquad 180 < \alpha < 225$$

Quad III

$$-\sqrt{\frac{3+1}{6}}$$

S	A
T	C

$$\cos\alpha = -\frac{\sqrt{6}}{3}$$

$$\sec\alpha = -\frac{3\cdot\sqrt{6}}{\sqrt{6}\cdot\sqrt{6}}$$

$$= \frac{\cancel{3}\sqrt{6}}{\cancel{6}2}$$

$$= -\frac{\sqrt{6}}{2}$$

$$-\sqrt{\frac{4}{6}}$$

$$-\sqrt{\frac{2}{3}}$$

$$-\frac{\sqrt{2}\cdot\sqrt{3}}{\sqrt{3}\cdot\sqrt{3}}$$

$$-\frac{\sqrt{6}}{3}$$

$$\cos\alpha = -\frac{\sqrt{6}}{3}$$

$$\sec\alpha = -\frac{\sqrt{6}}{2}$$

$$\sin\alpha = -\frac{\sqrt{3}}{3}$$

$$\csc\alpha = -\sqrt{3}$$

$$\tan\alpha = \frac{\sqrt{2}}{2}$$

$$\cot\alpha = \sqrt{2}$$

$$\sin\alpha = \pm\sqrt{\frac{1-\cos 2\alpha}{2}}$$

$$= \pm\sqrt{\frac{1-1/3}{2}\cdot 3}$$

$$= -\sqrt{\frac{3-1}{6}}$$

$$= -\sqrt{\frac{2}{6}}$$

$$= -\sqrt{\frac{1}{3}}$$

$$= -\frac{\sqrt{1}}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}$$

$$= -\frac{\sqrt{3}}{3}$$

$$\csc\alpha = -\frac{\sqrt{3}\cdot\sqrt{3}}{\sqrt{3}\sqrt{3}}$$

$$-\frac{\cancel{3}\sqrt{3}}{\cancel{3}}$$

$$-\sqrt{3}$$

$$\tan\alpha = \pm\sqrt{\frac{1-\cos 2\alpha}{1+\cos 2\alpha}} = \sqrt{\frac{(1-1/3)\cdot 3}{(1+1/3)\cdot 3}}$$

$$\sqrt{\frac{3-1}{3+1}} = \sqrt{\frac{2}{4}} = \frac{\sqrt{2}}{2}$$

#68 $\frac{\cos 2x}{2} = \cos x$

$x = 30°$

$\frac{\cos(2 \cdot 30)}{2} = \cos 30°$

$\frac{2 \cdot \frac{1}{2}}{2 \cdot 2} = \frac{\sqrt{3}}{2}$

$\frac{1}{4} \neq \frac{\sqrt{3}}{2}$

false when $x = 30°$

Not an identity