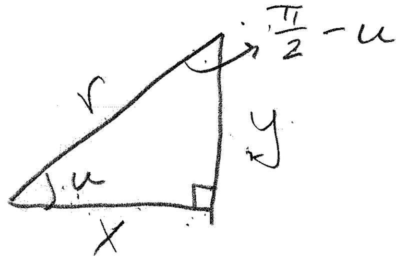
know 10/18/2012 - Sec 3.3 (cont.) and Sec 3.4 Math 1060

* Cofunction Identities



$\cos u = \frac{x}{r} = \sin\left(\frac{\pi}{2}-u\right)$

$\sin u = \cos\left(\frac{\pi}{2}-u\right)$

$\csc u = \sec\left(\frac{\pi}{2}-u\right)$

$\sec u = \csc\left(\frac{\pi}{2}-u\right)$

$\cot u = \tan\left(\frac{\pi}{2}-u\right)$

$\tan u = \cot\left(\frac{\pi}{2}-u\right)$

$\pi = u + \frac{\pi}{2} + ?$

$-\frac{\pi}{2} - u \quad -u \quad -\frac{\pi}{2}$

$\frac{\pi}{2} - u = ?$

$\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$

$\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

$\frac{3\pi}{3\cdot 2} - \frac{\pi}{6}$

$\frac{3\pi}{6} - \frac{\pi}{6} = \frac{2\pi}{6} = \frac{\pi}{3}$

$\cos\left(\frac{\pi}{2}-u\right) = \sin u$

(0,1)

$\cos\frac{\pi}{2}\cos u + \sin\frac{\pi}{2}\sin u$

$0 \cdot \cos u + 1 \cdot \sin u$

$0 + \sin u$

$\sin u = \sin u$ ✓

56

$$\underbrace{\cos(4)\cos(9)} + \cos(86)\cos(81)$$

cofunctions

$$\cos(4)\cos(9) + \sin(90-86)\sin(90-81)$$

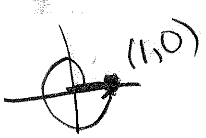
$$\cos(\overset{\alpha}{4})\cos(\overset{\beta}{9}) + \sin(\overset{\alpha}{4})\sin(\overset{\beta}{9})$$

$$\cos(4-9)$$

$$\cos(-5)$$

$$\boxed{\cos(5)}$$

#78 $\cos(\overset{\alpha}{\alpha} - \overset{\beta}{360°}) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$



$$\cos\alpha \, \underline{\cos 360°} + \sin\alpha \, \underline{\sin 360°}$$

$$\cos\alpha \cdot 1 + \underbrace{\sin\alpha \cdot 0}$$

$$\boxed{\cos\alpha}$$

Sec 3.4 Sum/Difference for Sine and Tangent

Sine of the Sum of Angles

$$\sin(\underbrace{\alpha+\beta}_{u}) = \cos\left(\frac{\pi}{2} - (\alpha+\beta)\right)$$

$$= \cos\left(\left(\frac{\pi}{2} - \alpha\right) - \beta\right)$$

$$= \cos\left(\frac{\pi}{2} - \alpha\right)\cos\beta + \sin\left(\frac{\pi}{2} - \alpha\right)\sin\beta$$

Given

$$\boxed{\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta}$$

$$\sin(\alpha - \beta)$$

$$\sin(\alpha + {}^{-}\beta) = \sin\alpha\cos(-\beta) + \cos\alpha\sin(-\beta)$$

$$= \sin\alpha\cos\beta - \cos\alpha\sin\beta$$

$$\sin(\overset{\alpha\quad\ \beta}{\overset{225 + 30}{255^\circ}}) = \sin 225\cos 30 + \cos 225\sin 30$$

$$-\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} + -\frac{\sqrt{2}}{2}\cdot\frac{1}{2}$$

$$-\frac{\sqrt{6}}{4} + -\frac{\sqrt{2}}{4}$$

$$\boxed{\frac{-\sqrt{6}-\sqrt{2}}{4}}$$

$255 =$

$360 - 45 = 305$

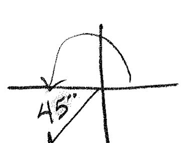
$360 - 90 = 270$

$270 - 15$

$270 - (45 - 30)$

$270 - 45 + 30$

$225 + 30$



tangent of Sum

$$\tan(\alpha+\beta) = \frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)}$$

$$= \frac{\sin\alpha\cos\beta + \cos\alpha\sin\beta \cdot \frac{1}{\cos\alpha\cos\beta}}{\cos\alpha\cos\beta - \sin\alpha\sin\beta \cdot \frac{1}{\cos\alpha\cos\beta}}$$

$$= \frac{\underbrace{\boxed{\frac{\sin\alpha}{\cos\alpha}}}_{}^{\tan\alpha}\frac{\cancel{\cos\beta}}{\cancel{\cos\beta}} + \frac{\cancel{\cos\alpha}}{\cancel{\cos\alpha}}\overbrace{\boxed{\frac{\sin\beta}{\cos\beta}}}^{\tan\beta}}{\frac{\cancel{\cos\alpha}\cancel{\cos\beta}}{\cancel{\cos\alpha}\cancel{\cos\beta}} - \boxed{\frac{\sin\alpha}{\cos\alpha}\frac{\sin\beta}{\cos\beta}}}$$

Given

$$* \quad \boxed{\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}}$$

$$\tan(\alpha-\beta)$$

$$\tan(\alpha+{}^-\beta) = \frac{\tan\alpha + \tan(-\beta)}{1 - \tan\alpha\tan(-\beta)}$$

$$= \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$$

-195

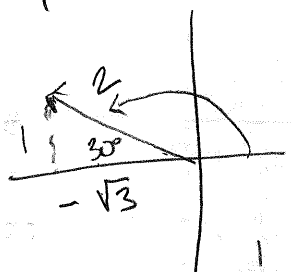
#15 $\tan\left(-\frac{13\pi}{12}\right) = -\tan\left(\frac{13\pi}{12}\right)$

$-\frac{13\cancel{\pi}}{\cancel{12}} \cdot \frac{\overset{15}{\cancel{180}}}{\cancel{\pi}} = -195$

$150 + 45 = 195$

$\frac{5\pi}{6} + \frac{\pi}{4} = \frac{13\pi}{12}$

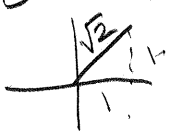
$\frac{3\pi}{4} + \frac{\pi}{3} = \frac{13\pi}{12}$



$\tan\frac{5\pi}{6} = \frac{1 \cdot \sqrt{3}}{-\sqrt{3} \cdot \sqrt{3}}$

$= -\frac{\sqrt{3}}{3}$

$\tan\frac{\pi}{4} = 1$



$= -\left[\tan\left(\frac{5\pi}{6} + \frac{\pi}{4}\right)\right]$

$= -\left[\frac{\tan\frac{5\pi}{6} + \tan\frac{\pi}{4}}{1 - \tan\frac{5\pi}{6}\tan\frac{\pi}{4}}\right]$

$-\left[\frac{-\frac{\sqrt{3}}{3} + 1}{1 - -\frac{\sqrt{3}}{3} \cdot 1}\right]$

$-\left[\frac{\left(-\frac{\sqrt{3}}{3} + 1\right)3}{\left(1 + \frac{\sqrt{3}}{3}\right)3}\right]$

$-\left[\frac{(-\sqrt{3} + 3)}{(3 + \sqrt{3})} \frac{(3 - \sqrt{3})}{(3 - \sqrt{3})}\right]$

$-\left[\frac{-3\sqrt{3} + 9 + (\sqrt{3})^2 - 3\sqrt{3}}{9 - \cancel{3\sqrt{3}} + \cancel{3\sqrt{3}} - (\sqrt{3})^2}\right]$

$-\left(\frac{\overset{2}{\cancel{12}} - \cancel{6}\sqrt{3}}{\cancel{6}}\right) = \boxed{-2 + \sqrt{3}}$