Math 1060 10/3/2012 - Sec 3.2

#16

Cot x in terms of Sec x

$1+\cot^2 x = \csc^2 x$ $\qquad \cot x = \frac{\cos x}{\sin x}$

$\sec x = \frac{1}{\cos x}$ $\qquad \cot x = \frac{1}{\tan x}$

$\tan^2 x + 1 = \sec^2 x$

$-1 \qquad -1$

$\sqrt{\tan^2 x} = \sqrt{\sec^2 x - 1}$

$\tan x = \pm\sqrt{\sec^2 x - 1}$

$$\cot x = \frac{1}{\pm\sqrt{\sec^2 x - 1}} = \boxed{\frac{\pm 1}{\sqrt{\sec^2 x - 1}}}$$

#10

$$\frac{(\cos\alpha\tan\alpha + 1)(\sin\alpha - 1)}{\cos^2\alpha}$$

$$\frac{\left(\cancel{\cos\alpha}\cdot\frac{\sin\alpha}{\cancel{\cos\alpha}} + 1\right)(\sin\alpha - 1)}{\cos^2\alpha}$$

$\sin^2\alpha + \cos^2\alpha = 1$

$-\cos^2\alpha \qquad -\cos^2\alpha$

$\sin^2\alpha = 1 - \cos^2\alpha$

$-1 \qquad -1$

$\sin^2\alpha - 1 = -\cos^2\alpha$

$$\frac{(\sin\alpha+1)(\sin\alpha-1)}{\cos^2\alpha} = \frac{\sin^2\alpha - 1}{\cos^2\alpha} = \frac{-\cos^2\alpha}{\cos^2\alpha} = -1$$

Sec 3.2 Verifying Identities

$\sin^2 x + \cos^2 x = 1$ $\qquad$ $\tan^2 x + 1 = \sec^2 x$

Ex: $1 - \sec x \csc x \tan x = -\tan^2 x$

$$1 - \frac{1}{\cos x} \cdot \frac{1}{\sin x} \cdot \frac{\sin x}{\cos x}$$

$$1 - \frac{1}{\cos^2 x}$$

$\left(\frac{1}{\cos x}\right)^2 = (\sec x)^2$

$\frac{1}{\cos^2 x} = \sec^2 x$

$$1 - \sec^2 x$$

$$1 - (\tan^2 x + 1)$$

$$1 - \tan^2 x - 1$$

$$-\tan^2 x = -\tan^2 x \quad \checkmark$$

Algebra skills to Remember

$$(a+b)(a-b) = a^2 - b^2$$

$$\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b}$$

$\longrightarrow$

$\longleftarrow$

$$x^2 + 3x + 2$$

$$(x+2)(x+1)$$

$$\tan^2\theta + 3\tan\theta + 2$$

$$(\tan\theta + 2)(\tan\theta + 1)$$

Verify: $\dfrac{\cos\theta}{1-\sin\theta} = \dfrac{1+\sin\theta}{\cos\theta}$

$$\frac{(1+\sin\theta)}{(1+\sin\theta)}\cdot\frac{\cos\theta}{(1-\sin\theta)}$$

$$\frac{\cos\theta(1+\sin\theta)}{1-\sin^2\theta}$$

$\cos^2\theta + \sin^2\theta = 1$
$\cos^2\theta = 1 - \sin^2\theta$ ← $1-\sin^2\theta$

$$\frac{\cancel{\cos\theta}(1+\sin\theta)}{\cos^{\cancel{2}}\theta}$$

$$\frac{1+\sin\theta}{\cos\theta} = \frac{1+\sin\theta}{\cos\theta}$$ ✓

Strategies

1. Work on one side keeping in mind the other side as your goal
2. Sometimes it helps to rewrite in terms of sines and cosines
3. Remember you can multiply top and bottom of a fraction by a common value to help
4. Sometimes splitting apart a fraction helps. $\frac{a+c}{b} = \frac{a}{b} + \frac{c}{b}$
5. Sometimes combining into a single fraction helps.
6. Sometimes you have to work on both sides (seperately) to get to a third expression
7. Try, Try, again

Verify: $\dfrac{\sec x - \cos x}{\cos x} = \tan^2 x$

$$\frac{\sec x}{\cos x} - \frac{\cos x}{\cos x}$$

$$\frac{\sec x}{\cos x} - 1$$

$\sec x \cdot \frac{1}{\cos x}$

$\frac{1}{\cos x} \cdot \frac{1}{\cos x}$

$$\frac{\frac{1}{\cos x}}{\cos x} - 1$$

$$\frac{1}{\cos x} \cdot \frac{1}{\cos x} - 1$$

$$\frac{1}{\cos^2 x} - 1$$

$$\sec^2 x - 1$$

$$\tan^2 x = \tan^2 x \checkmark$$