10/1/12 - Sec 3.1

Math 1060

#44

$V_0 = 3$ in/sec ← initial velocity

Start $x_0 = 1$ in below equilibrium

$\omega = \sqrt{3}$

equation at t

$t = 2$

Pg 157 $X = \frac{V_0}{\omega}\sin(\omega t) + X_0\cos(\omega t)$

$\frac{3\cdot\sqrt{3}}{\sqrt{3}\cdot\sqrt{3}} = \frac{\cancel{3}\sqrt{3}}{\cancel{3}}$ $X = \frac{\sqrt{3}\,3}{\sqrt{3}\cdot\sqrt{3}}\sin(\sqrt{3}\,t) + 1\cos(\sqrt{3}\,t)$

$$\boxed{X = \sqrt{3}\sin(\sqrt{3}t) + \cos(\sqrt{3}\,t)}$$

$X = \sqrt{3}\sin(2\sqrt{3}) + \cos(2\sqrt{3})$

Amp: 2

Period: 3.75

Sec 3.1 Basic Identities

An <u>identity</u> is an equation that is satisfied by every number for which both sides are defined.

formula: $P = 2L + 2W$

Know
★ <u>Reciprocal Identities</u>

$\sin x = \frac{1}{\csc x}$ $\quad$ $\cos x = \frac{1}{\sec x}$ $\quad$ $\tan x = \frac{1}{\cot x}$

$\csc x = \frac{1}{\sin x}$ $\quad$ $\sec x = \frac{1}{\cos x}$ $\quad$ $\cot x = \frac{1}{\tan x}$

Recall
$\sin x = \frac{opp}{hyp}$ $\quad$ $\cos x = \frac{adj}{hyp}$ $\quad$ $\tan x = \frac{opp}{adj}$

$\cot x = \frac{adj}{opp}$

Know
★ $\tan x = \frac{\sin x}{\cos x}$ $\quad$ $\cot x = \frac{\cos x}{\sin x}$

why $\frac{opp}{adj} = \frac{opp/hyp}{adj/hyp} = \frac{opp}{\cancel{hyp}} \cdot \frac{\cancel{hyp}}{adj} = \frac{opp}{adj}$

Recall The Fundamental Identity of Trig

$$\sin^2\theta + \cos^2\theta = 1$$

divide by $\sin^2\theta$

$$\frac{\sin^2\theta}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta} = \frac{1^2}{\sin^2\theta}$$

$$1 + \left(\frac{\cos\theta}{\sin\theta}\right)^2 = \left(\frac{1}{\sin\theta}\right)^2$$

$$1 + (\cot\theta)^2 = (\csc\theta)^2$$

$$1 + \cot^2\theta = \csc^2\theta$$

divide by $\cos^2\theta$

$$\frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$$

$$\tan^2\theta + 1 = \sec^2\theta$$

know

* Pythagorean Identities *

$$\sin^2 x + \cos^2 x = 1$$

$$\tan^2 x + 1 = \sec^2 x$$

$$1 + \cot^2 x = \csc^2 x$$

Simplify

$$\frac{\cot x}{\csc x} = \frac{\cos x/\sin x}{1/\sin x} = \frac{\cos x}{\cancel{\sin x}} \cdot \frac{\cancel{\sin x}}{1}$$

$$= \cos x$$

helpful when simplifying

- Rewrite in terms of sines and cosines
- multiply things out to see if it's helpful

Ex: $(1+\sin x)(1-\sin x)$

$$1 - \cancel{\sin x} + \cancel{\sin x} - \sin^2 x$$

$$1 - \sin^2 x$$

$$\boxed{\cos^2 x}$$

$$\sin^2 x + \cos^2 x = 1$$
$$-\sin^2 x \qquad\qquad -\sin^2 x$$
$$\cos^2 x = 1 - \sin^2 x$$

#65 $\sin^4 x - \cos^4 x \Rightarrow a^4 - b^4 = (a^2)^2 - (b^2)^2$

$$(\sin^2 x + \cos^2 x)(\sin^2 x - \cos^2 x) \quad (a^2+b^2)(a^2-b^2)$$

$$1(\sin^2 x - \cos^2 x)$$

$$\boxed{\sin^2 x - \cos^2 x} = (\sin x + \cos x)(\sin x + \cos x)$$

$$\sin^2 x + \cos^2 x = 1$$

$$\cos^2 x = 1 - \sin^2 x$$

$$\cos x = \pm\sqrt{1-\sin^2 x}$$

If $\tan\alpha = \frac{1}{2}$ and $\pi < \alpha < \frac{3\pi}{2}$

use identities to find the remaining 5 trig values

$\cot\alpha = 2$

$\sec\alpha = -\sqrt{5}/2$

$\cos\alpha = -2\sqrt{5}/5$

$$\cot\alpha = \frac{1}{\tan\alpha} = \frac{1}{1/2} = 2$$

$$1 + \cot^2\alpha = \csc^2\alpha$$

$$\tan^2\alpha + 1 = \sec^2\alpha$$

$$\left(\tfrac{1}{2}\right)^2 + 1 = \sec^2\alpha$$

$$\frac{1}{4} + 1 = \sec^2\alpha$$

$$\pm\sqrt{\frac{5}{4}} = \sqrt{\sec^2\alpha}$$

$$-\frac{\sqrt{5}}{2} = \sec\alpha$$

$$\cos\alpha = \frac{1}{\sec\alpha} = -\frac{2\cdot\sqrt{5}}{\sqrt{5}\cdot\sqrt{5}}$$

$$= \frac{-2\sqrt{5}}{5}$$

An <u>even</u> function has y-axis symmetry and $f(-x) = f(x)$

An <u>odd</u> function has origin symmetry and $f(-x) = -f(x)$

* <u>Even</u>

$\cos(-x) = \cos(x)$, $\sec(-x) = \sec(x)$

* <u>odd</u>

$\sin(-x) = -\sin(x)$, $\csc(-x) = -\csc(x)$

$\tan(-x) = -\tan(x)$, $\cot(-x) = -\cot(x)$

If an equation is <u>Not</u> an identity

$\cos(3t) \stackrel{?}{=} 3\cos t$ test at $t = \pi/3$

$\cos(3 \cdot \frac{\pi}{3}) = 3\cos(\pi/3)$

$\cos \pi = 3 \cdot \frac{1}{2}$

$-1 \neq 3/2$

Not an identity