

10/8/2012 - Sec 7.1

Math 1010

Natural #'s : $\{1, 2, 3, \dots\}$

Whole #'s : $\{0, 1, 2, \dots\}$

Integers #'s : $\{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$

Rational #'s : $\left\{ \frac{p}{q} \mid p, q \text{ were both integers} \right\}$

↑

means fractions

Expression: doesn't have an Equal sign

Sec 7.1 Rational Expressions and Functions
Multiplying and dividing

Rational Expression: is a function
defined by a quotient of polynomials

Ex: $f(x) = \frac{2}{x-5}$, $\frac{4x+1}{x^2+3x+2}$

The Domain of a Rational Function
is all Real Numbers except those
that make the denominator equal zero

Ex: $f(x) = \frac{2}{x-5}$ $D: \{x \mid x \neq 5\}$ $\begin{array}{r} x-5=0 \\ +5 \quad +5 \\ \hline x=5 \end{array}$

$$\frac{4x+1}{x^2+3x+2}$$

$$D: \{x \mid x \neq -2, -1\}$$

$$x^2+3x+2=0$$

$$(x+2)(x+1)=0$$

$$\begin{array}{cc} x+2=0 & x+1=0 \\ \begin{array}{cc} -2 & -2 \end{array} & \begin{array}{cc} -1 & -1 \end{array} \\ x=-2 & x=-1 \end{array}$$

Find Domain for $h(x) = \frac{8x+2}{3}$

$$D: \{x \mid x \text{ is a Real Number}\}$$

or $(-\infty, \infty)$

Lowest terms

$$\frac{10}{15} = \frac{2}{3} \checkmark$$

$$\frac{a \cdot c}{b \cdot c} = \frac{a}{b}$$

$$\frac{2 \cdot \cancel{5}}{3 \cdot \cancel{5}}$$

So to write an expression in lowest terms factor the numerator and denominator then divide out common factors

$$\frac{(x+5)(\cancel{x+2})}{(\cancel{x+2})(x-3)} = \frac{\overset{\text{term}}{\cancel{(x+5)}}}{\underset{\text{term}}{(x-3)}} = \frac{x+5}{x-3}$$

Remember you can only divide out common factors not common terms.

$$\frac{\overset{\text{simplified}}{\cancel{8+k}}}{16} \leftarrow \text{factor}$$

$$\frac{10+2k}{16} = \frac{\overset{\text{factor}}{\cancel{2}}(5+k)}{\cancel{16} 8} = \frac{5+k}{8}$$

Watch for -1

$$\frac{a-10}{10-a} = \frac{(a-10)}{-1(-10+a)} = \frac{(a-10)}{-1(a-10)}$$

$$= \frac{\cancel{(a-10)}}{-1\cancel{(a-10)}}$$

$$= \frac{1}{-1}$$

$$= \boxed{-1}$$

Multiplying Rational Expressions

Recall: $\frac{a \rightarrow c}{b \rightarrow d} = \frac{ac}{bd}$

$$\frac{1}{3} \cdot \frac{6}{7} = \frac{6}{3 \cdot 7} = \frac{2 \cdot \cancel{3}}{\cancel{3} \cdot 7} = \frac{2}{7}$$

$$\begin{aligned} \text{Ex: } \frac{(5p-5)}{p} \cdot \frac{3p^2}{(10p-10)} &= \frac{(5p-5)3p^2}{p(10p-10)} \checkmark \\ &= \frac{\cancel{5}(p-1)3p^2}{p \cdot \cancel{10}(p-1)} \\ &= \boxed{\frac{3p}{2}} \end{aligned}$$

$$\begin{aligned} \text{Ex: } \frac{c^2+2c}{c^2-4} \cdot \frac{c^2-4c+4}{c^2-c} & \\ \frac{\cancel{c}(c+2)}{(c-2)(\cancel{c}+2)} \cdot \frac{(c-2)^2}{\cancel{c}(c-1)} & \\ \frac{(c-2)}{(c-1)} &= \boxed{\frac{c-2}{c-1}} \end{aligned}$$

$$\text{Ex: } \frac{x^2 + 2x}{x+1} \cdot \frac{x^2 - 1}{x^3 + x^2} = \frac{(x+2)(x-1)}{x(x+1)}$$

$$\frac{\cancel{x}(x+2) \cdot (x-1)\cancel{(x+1)}}{(x+1) \cdot x^2 \cancel{(x+1)}}$$

$$= \frac{(x+2)(x-1)}{x(x+1)} \checkmark$$

Reciprocals: $\frac{3}{2}, \frac{2}{3}$

$4, \frac{1}{4}$

two expressions are reciprocals if their product is 1

$$\frac{3}{2} \div \frac{4}{5} = \frac{3 \rightarrow 5}{2 \rightarrow 4} = \frac{3 \cdot 5}{2 \cdot 4} = \boxed{\frac{15}{8}}$$

$$\text{Ex: } \frac{5p+2}{6} \div \frac{15p+6}{5}$$

$$\frac{5p+2}{6} \cdot \frac{5}{15p+6} = \frac{\cancel{(5p+2)} \cdot 5}{6 \cdot 3 \cancel{(5p+2)}} = \boxed{\frac{5}{18}}$$

$$\#78 \quad (y^2-4) \div \frac{2-y}{8y}$$

$$\frac{(y^2-4)}{1} \cdot \frac{8y}{(2-y)}$$

$$-(-2+y)$$

$$\frac{(y+2)(y-2)}{1} \cdot \frac{8y}{-(y-2)}$$

$$= \frac{8y(y+2)}{-1}$$

$$= \boxed{-8y(y+2)}$$