Sec 5.1 Integer Exponents and Scientific Notation

Recall

$$\underbrace{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}_{\text{5 of them}} = 2^{5}$$

(5 ← exponent; 2 ↑ base)

Read "2 to the 5th power"

$$3^{4} = 3 \cdot 3 \cdot 3 \cdot 3 = 81$$

x^{y} or y^{x}

Rules for Exponents

- **Product Rule:** if bases the same, add exponents

$$\star \quad a^{m} \cdot a^{n} = a^{m+n}$$

why?

$$2^{3} \cdot 2^{6} = 2^{3+6} = 2^{9}$$

$$\underbrace{\underbrace{2 \cdot 2 \cdot 2}_{3} \cdot \underbrace{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}_{6}}_{9} = 2^{9}$$

Quotient Rule: bases same when dividing, subtract exponents

$$\frac{a^m}{a^n} = a^{m-n}$$

Why?

$$\frac{2^7}{2^3} = 2^{7-3} = 2^4$$

$$\frac{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}{2 \cdot 2 \cdot 2} = 2^4$$

Ex: $\frac{t^5}{t^4} = t^{5-4} = t^1 = t$

$$\frac{d^4}{d^7} = d^{4-7} = d^{-3}$$

$$\frac{d \cdot d \cdot d \cdot d}{d \cdot d \cdot d \cdot d \cdot d \cdot d \cdot d} = \frac{1}{d^3}$$

$$d^{-3} = \frac{1}{d^3}$$

Negative exponents

$$a^{-n} = \frac{1}{a^n} \quad \text{and} \quad \frac{1}{a^{-n}} = a^n$$

Ex:

$$\frac{1}{2^{-4}} = 2^4$$

$$\frac{3^1 x^{-2}}{1} = \frac{1}{3x^2} \qquad (3x)^{-2} = \frac{1}{(3x)^2}$$

$$= \frac{3}{x^2}$$

$$\frac{x^{-2}}{y^{-5}} = \boxed{\frac{y^5}{x^2}}$$

Side Example

$$\frac{y^3}{y^5} = \frac{1}{y^2}$$

Ex: $\dfrac{x^2 y^4}{x^{-3} y^2}$

option 1 (Product, Quotient)

$$\frac{x^3 \cdot x^2 \cdot y^4}{y^2} = \boxed{x^5 y^2}$$

option 2 (Quotient Rule)

$$\boxed{\frac{x^2}{x^{-3}} \quad \frac{y^4}{y^2}} = x^{2+3} \; y^{4-2} = \boxed{x^5 y^2}$$

$2^0 = 1$

$a^0 = 1$

$0^1 = 0$

$0^2 = 0$

$0^n = 0$

0^0 is undefined

Zero Exponent Rule:

$a^0 = 1$ if $a \neq 0$

~~x^0~~

$$\frac{x^4}{x^4} = x^{4-4} = \boxed{x^0} = \boxed{1}$$

$$\frac{\cancel{x}\cdot\cancel{x}\cdot\cancel{x}\cdot\cancel{x}}{\cancel{x}\cdot\cancel{x}\cdot\cancel{x}\cdot\cancel{x}} = \frac{1}{1} = 1$$

Ex: $6^0 = 1$

$-6^0 = -1$

$(-6)^0 = 1$

$10^0 - 9^0 = 0$

$1 - 1 = 0$ ✓

Power Rule

★ $(a^m)^n = a^{mn}$, $(ab)^m = a^m b^m$, $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

Why?

$$(2^4)^3 = 2^{4\cdot 3} = 2^{12}$$

$$= 2^4 \cdot 2^4 \cdot 2^4 = 2^{4+4+4} = 2^{4\cdot 3} = 2^{12}$$

$$(ab)^3 = a^3 b^3$$

$$= (ab)(ab)(ab)$$

$$= a \cdot a \cdot a \cdot b \cdot b \cdot b$$

$$= a^3 b^3$$

$$\left(\frac{a}{b}\right)^3 = \frac{a^3}{b^3}$$

$$\frac{2}{5} \cdot \frac{6}{7} = \frac{12}{35}$$

$$= \left(\frac{a}{b}\right)\left(\frac{a}{b}\right)\left(\frac{a}{b}\right)$$

$$= \frac{a \cdot a \cdot a}{b \cdot b \cdot b} = \frac{a^3}{b^3} \checkmark$$

Not an Exponent Rule

$(x+y)^2 \neq x^2 + y^2$

what the Rule is

$(xy)^2 = x^2 y^2$

Ex:

$(3^1 x^2 y^1)^5$ = $3^5 x^{10} y^5$ ← Vote 1

↓ Vote 2

$243 x^{10} y^5$ ←

$(3^1)^5 (x^2)^5 (y^1)^5 = 3^{1.5} x^{2.5} y^{1.5}$

$x^2 \cdot x^5$ $\quad$ $3^5 x^{10} y^5$

$(a^m)^n = a^{m \cdot n}$

$(3x^2y)(3x^2y) = 3 \cdot 3 \cdot x^2 \cdot x^2 \cdot y^1 \cdot y^1$

$3^2 x^{2+2} \cdot y^{1+1}$

$9x^4y^2$

another Negative exponent Rule

$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$

why?

$\left(\frac{a}{b}\right)^{-n} = \frac{a^{-n}}{b^{-n}} = \frac{b^n}{a^n} = \left(\frac{b}{a}\right)^n$