9/13/2012 - Sec 4.1 (cont.)
Sec 4.2

4.1

#33 $y = 2x$ ←

$4x - 2y = 0$

$4x - 2(2x) = 0$

$4x - 4x = 0$

$0 = 0$ ← true

true statement at the end means infinitely many solutions

$\{(x,y) \mid y = 2x\}$

or

$\{(x,y) \mid 2x - y = 0\}$

#18 $2x + y = 4 \Rightarrow y = -2x + 4$

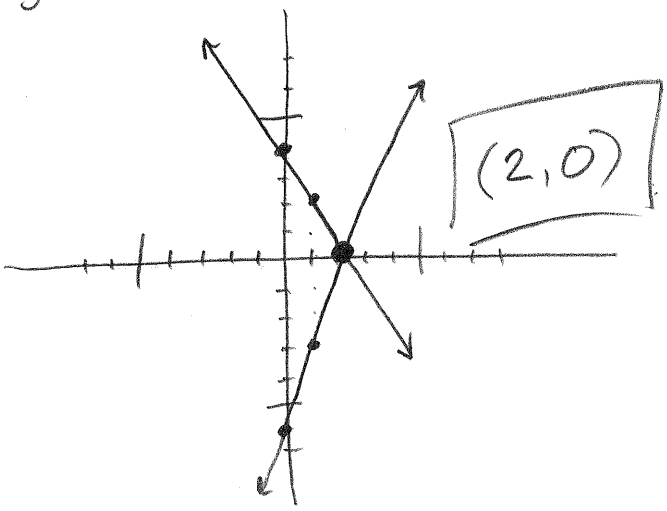
$3x - y = 6 \Rightarrow y = 3x - 6$,

$\frac{-y}{-1} = \frac{-3x}{-1} + \frac{6}{-1}$

Check

$2(2) + 0 = 4$ ✓

$3(2) - 0 = 6$ ✓



Ex: of NO solution

$(-2x + 5y = 6) \cdot 3$

$6x - 15y = 4$

$-6x + 15y = 18$

$6x - 15y = 4$

$0 \neq 22$ ← false

Solution: $\emptyset$

or

NO solution

If when substitute or eliminate and I'm left with NO variables and

1. a true statement: infinitely many solutions
2. a false statement: NO solution

Use the slope and y-int to decide how many solutions

- different slopes: exactly 1 solution
- same slope, different y-int (parallel lines) NO solution
- same slope, same y-int (same line) infinitely many solutions

Sec 4.2 Systems of Linear equations in three variables

Ex: Linear Eq in 3 Variables

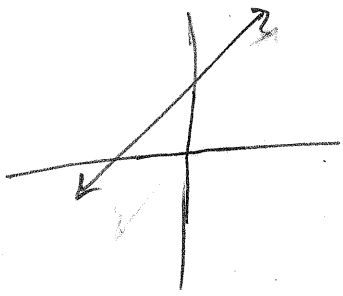
$$4x + 8y + z = 2$$

ordered triples: (x, y, z)

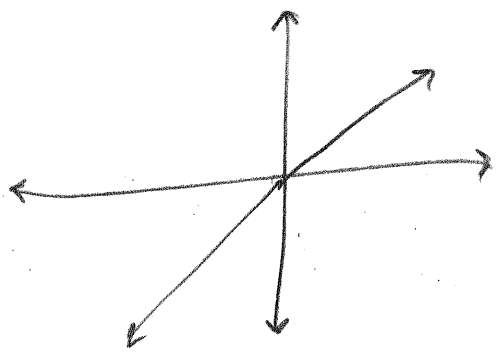
Solution: $(-2, 1, 2)$

$$4(-2) + 8(1) + (2) = 2$$
$$-8 + 8 + 2 = 2$$
$$2 = 2 \checkmark$$

2-variables



3-variable



Possible Solution Situations

Case 1 the 3 planes meet at a single common point
(a single point answer)

Case 2 the 3 planes meet at a common line
(infinitely many solutions described by equation of line)

Case 3 the three planes coincide
(infinitely many solution described by equation of the plane)

Case 4 the 3 planes have no points in common
(no solution)

How to Solve (using Elimination)

1st pick a variable to eliminate and an equation to do it with

2nd use my picked equation with the other two equations to eliminate the variable chosen

3rd the two Resulting equations should have 2 variables only, Solve for those two variables (just like in Sec 4.1)

4th Use the two answers to find the value of the third variable

5th Check the solution in each of the original equations

Ex: $\begin{cases} x - y + 2z = 1 \\ 3x + 2y + 7z = 8 \\ -3x - 4y + 9z = -10 \end{cases}$ ← ↓

Eliminate x
Use Eq 1

$-3(x - y + 2z = 1) \Rightarrow -3x + 3y - 6z = -3$

$3x + 2y + 7z = 8$

$5y + z = 5$ *

$3(x - y + 2z = 1) \Rightarrow 3x - 3y + 6z = 3$

$-3x - 4y + 9z = -10$

$-7y + 15z = -7$ *

$5y + z = 5 \Rightarrow z = -5y + 5$

$-7y + 15z = -7$

$-7y + 15(-5y + 5) = -7$

$-7y - 75y + 75 = -7$

$-75 \quad -75$

$\frac{-82y}{-82} = \frac{-82}{-82}$

$y = 1$

$z = -5(1) + 5$

$z = -5 + 5$

$z = 0$

$x - y + 2z = 1$

$x - (1) + 2(0) = 1$

$x - 1 = 1$

$x = 2$ $(2, 1, 0)$

Ex: $x - y = 6$
$2y + 5z = 1$
$3x - 4z = 8$

$\Rightarrow$

$x - y = 6$
$2y + 5z = 1$
$3x + 4z = 8$

If when eliminating we get a false statement: we're done!
answer is NO solution

If when eliminating we get a true statement (both times!): infinitely many solutions

when true statements occur going from 3 variables to 2 → Solution is the entire plane

when true statement occurs solving 2eq 2Var → Solution is the line

#38 $2x - 8y + 2z = -10$

eliminate ys → $-x + 4y - z = 5$

$\frac{1}{8}x - \frac{1}{2}y + \frac{1}{8}z = -\frac{5}{8}$

$2x - 8y + 2z = -10$
$2(-x + 4y - z = 5) \Rightarrow -2x + 8y - 2z = 10$

$0 = 0$

$$-x + 4y - z = 5$$

$$8\left(\frac{1}{8}x - \frac{1}{2}y + \frac{1}{8}z = -\frac{5}{8}\right) \Rightarrow \begin{array}{r} -x + 4y - z = 5 \\ \hline x - 4y + z = -5 \end{array}$$

$$0 = 0$$

$$\{(x, y, z) \mid -x + 4y - z = 5\}$$