

Story 6. A Tight Weave



Sir Pinsky, a famous name in carpets, has a worldwide reputation for pushing the limits of the art of floor covering. The fashion world stands agog at the clean lines and uncanny coherence of his purple and gold creations. Some call him square because his designs so richly employ that quaint quadrilateral. But squares in the hands of a master can create textures beyond the weavers' world, although not beyond human imagination.

One day Sir Pinsky began a creation with, as always, a perfect, purple square. However, one square seemed too plain, so in the exact center of it he added a gold square. He saw that the central square implicitly defined eight purple squares surrounding it. As he pondered, he realized that those eight purple squares were identical to his original large square except for two things: (1) each was one-third the size of the whole square; and (2) none of them had a gold square in its center.

He wondered whether he could further modify his design so that each of the eight small squares would replicate the entire design except for being one-third its size. After much thought, he solved this puzzle and created a design with which his name is associated. Can you sketch and describe his design? Create this design in stages, adding more gold squares at each stage.

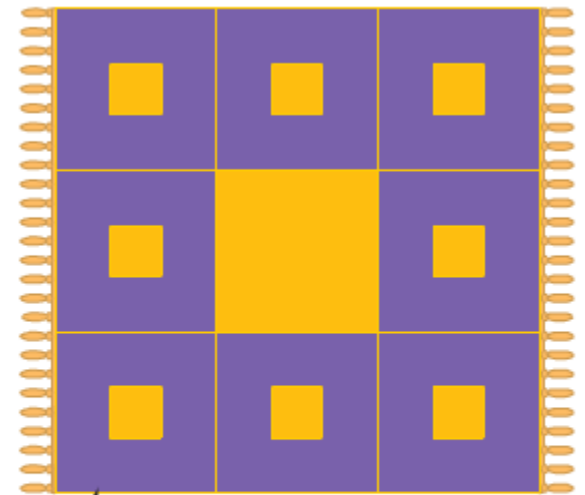
Suppose the original square rug is 1 yard by 1 yard. How much gold material would be needed for the second stage? How much for the third stage? Continue computing the area of the gold squares at various stages of the process, and then guess how much gold material will be needed to create the final floor covering. The answer is surprising.

Though our carpet designer is thoroughly modern in all ways, the source of his inspiration is ancient. In Chapter 6, "Fractals and Chaos," we will see an example of this style in the 19th-century Buddhist tapestry, Vaishravana Mandala.

Story 6. A Tight Weave

Nudges:

- Consider a purple square that has a smaller gold square in its center. How do each of the eight surrounding squares differ from the whole picture? They are much the same except the whole picture has a gold square in the middle, and each of the eight surrounding squares is solid purple. How could you modify those eight one-third-size surrounding squares to make them look like smaller copies of the entire picture?



We want each square to look like a smaller copy of the entire rug.

Story 6. A Tight Weave

Nudges:

- Now let's ask the question again: "In the picture you now have, is each of the eight one-third-size squares identical to smaller copies of the whole picture?" No. How would you modify each one-third-size square-with-a-gold-center to make it identical to the whole new figure? Are you done?
- Draw several steps of this repetitive process. At each stage, add up the areas of all the gold squares. When should you stop this process?