5.3 Polynomial Functions, Graphs, and Composition

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Objectives:

- Recognize and evaluate polynomial functions.
- Use a polynomial function to model data.
- Add and subtract polynomial functions.
- Find the composition of functions.
- Graph basic polynomial functions.

Polynomial Function

- A **polynomial function of degree n** is defined by

For all real numbers $a_n, a_{n-1}, \ldots, a_1$, and a_0 where $a \neq 0$ and n is a whole number.

Example 1

- Evaluate the following polynomial functions.
  - Let $f(x) = 4x^2 - x + 5$. Find $f(2)$.
  - Let $f(x) = x^3 - x^2 + 7$. Find $f(-3)$.

Use a Polynomial Function to Model Data

- The number of students enrolled in public schools (*grades pre-K-12*) in the United States during the years 1990 through 2006 can be modeled by the polynomial function defined by

$$P(x) = -0.01774x^2 + 0.7871x + 41.26,$$

where $x = 0$ corresponds to the year 1990, $x = 1$ corresponds to 1991, and so on, and $P(x)$ is in millions. Use this function to approximate the number of public school students in 2006.

Adding & Subtracting Polynomial Functions

- If $f(x)$ and $g(x)$ define functions, then

The domain of the new function is the intersection of the domains of $f(x)$ and $g(x)$.

Example 2

- Find each of the following for the polynomial functions defined by

  $\boldsymbol{f(x) = 3x^2 + 8x - 6}$ **and** $\boldsymbol{g(x) = -4x^2 + 4x - 8}$

  - $(f + g)(x)$

  - $(f - g)(x)$

Example 3

- Find each of the following for the polynomial functions defined by

$$\boldsymbol{f(x) = 18x^2 - 24x} \quad \textbf{and} \quad \boldsymbol{g(x) = 3x}$$

  - $(f + g)(x)$

  - $(f + g)(-1)$

Example 4

- Find each of the following for the polynomial functions defined by

$$\boldsymbol{f(x) = 18x^2 - 24x} \quad \textbf{and} \quad \boldsymbol{g(x) = 3x}$$

  - $(f - g)(x)$

  - $(f - g)(1)$

Composition of Functions

- A $40 pair of blue jeans is on sale for 25% off. If you purchase the jeans before noon, the retailer offers an additional 10% off. What is the final sale price of the blue jeans?

Example 5

- Let $f(x) = x - 4$ and $g(x) = x^2$, find $(f \circ g)(3)$.

Example 6

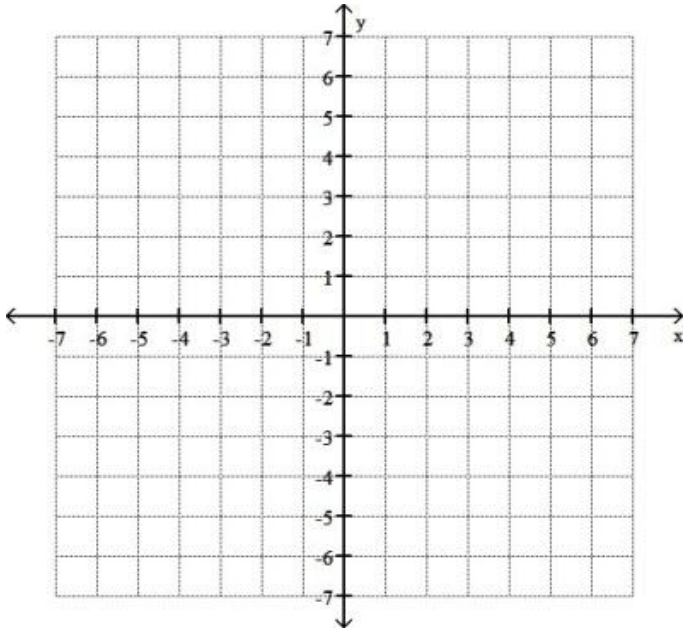
- Let $\boldsymbol{f(x) = 4x - 1}$ and $\boldsymbol{g(x) = x^2 + 5}$, find the following.
  - $(f \circ g)(2)$
  - $(g \circ f)(2)$

Example 6 *(continued)*

- Let $\boldsymbol{f(x) = 4x - 1}$ and $\boldsymbol{g(x) = x^2 + 5}$, find the following.
  - $(f \circ g)(x)$

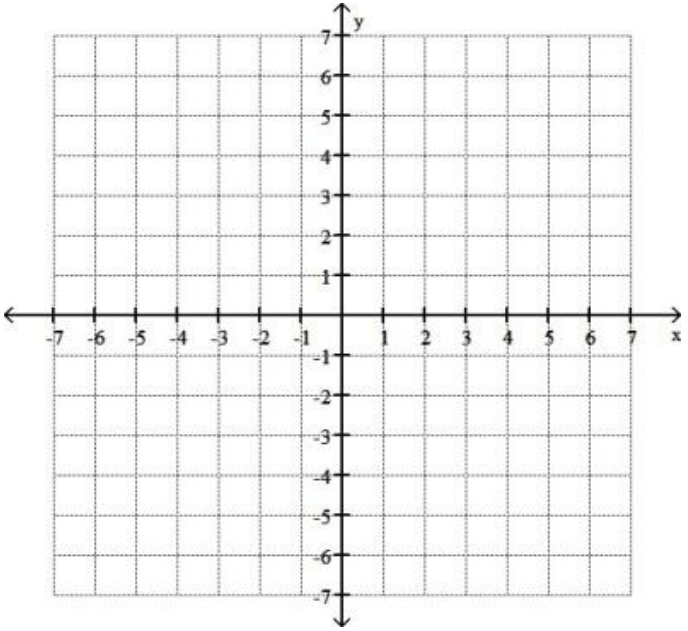
Graph the Identity Function

- Graph the identity function $f(x) = x$, then state the domain and range.



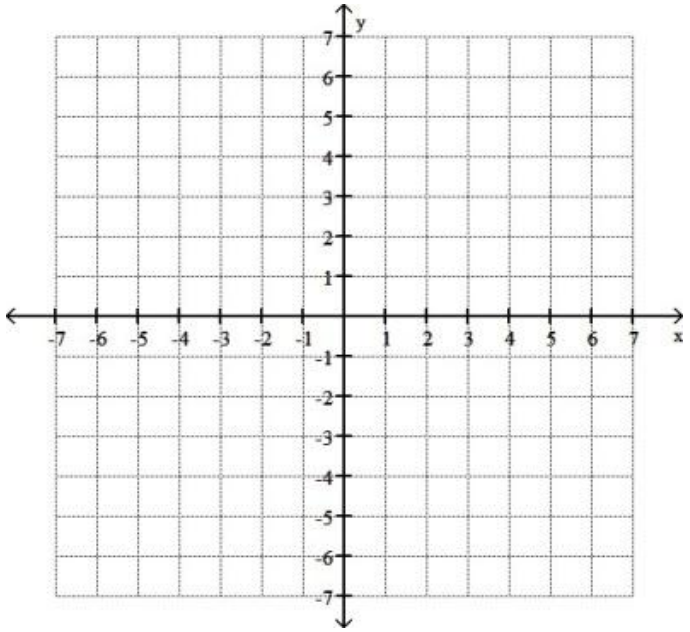
Graph the Quadratic Function

- Graph the quadratic or squaring function $f(x) = x^2$, then state the domain and range.



Graph the Cubic Function

- Graph the cubic or cubing function $f(x) = x^3$, then state the domain and range.



Graph the Polynomial Function

- Graph the function $f(x) = -x^2 + 4$, then state the domain and range.