3.5 Introductions to Relations and Functions

Written by: Cindy Alder

Objectives:

- Distinguish between independent and dependent variables.
- Define and identify relations and functions.
- Find the domain and range.
- Identify functions defined by graphs and equations.

INDEPENDENT AND DEPENDENT VARIABLES

- We often describe one quantity in terms of another. Consider the following:
  - The amount of your paycheck if you are paid hourly depends on the number of hours you worked.

  - The cost at the gas stations depends on the number of gallons of gas you pumped into your car.

  - The distance traveled by a car moving at a constant speed depends on the time traveled.

- We can use ordered pairs to represent these corresponding quantities.
  - For example: the relationship between the amount of your paycheck and the hours worked can be written where the first number represents the hours worked and the second number represents the paycheck amount in dollars.
    - The ordered pair (**5**, **40**) indicates that when you work 5 hours your paycheck is $40.
    - The ordered pair (**8**, **64**) indicates that when you work 8 hours your paycheck is $64.
  - Since the amount of your paycheck *depends* on the number of hours worked, your paycheck amount is called the ***dependent variable***, and the number of hours worked is called the ***independent variable***.

- If the values of the variable y depends on the value of the variable x, then y is the **dependent variable** and x is the **independent variable**

$$(x, y)$$

RELATIONS AND FUNCTIONS

Relation
A relation

Function
A function is a relation in which,

- Determine whether each relation defines a function.
  - $A = \{(1,1), (-2,4), (3,-1)\}$
    -
  - $B = \{(-2,-1), (-1,0), (0,1), (1,2), (2,2)\}$
    -

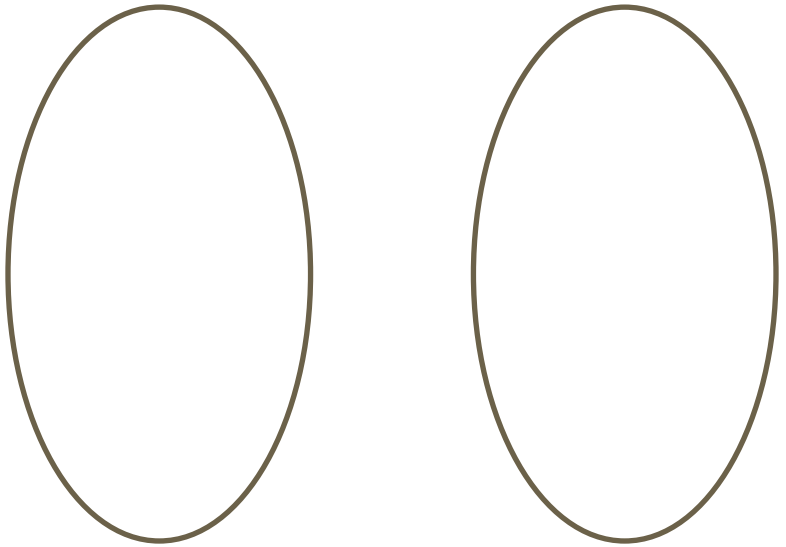
      (The last two ordered pairs have the same y-value. This does not violate the definition of a function.)
  - $C = \{(-4,1), (-2,-1), (-2,0)\}$
    -

Relations and Functions can be Defined Several Ways

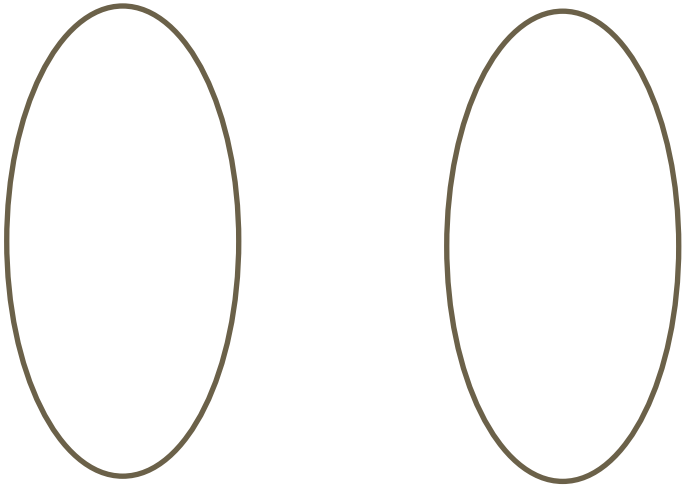
- As a set of ordered pairs.
- As a correspondence or mapping.
- As a table.
- As a graph.
- As an equation. (or rule)

Correspondence or Mapping

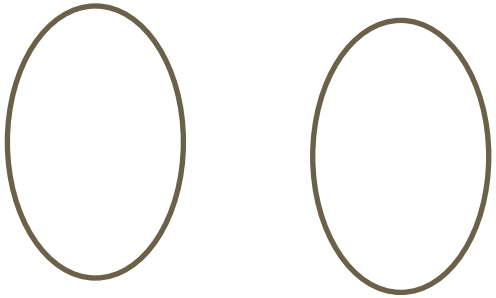
$$A = \{(1,1),(-2,4),(3,-1)\}$$



$$B = \{(-2,-1),(-1,0),(0,1),(1,2),(2,2)\}$$



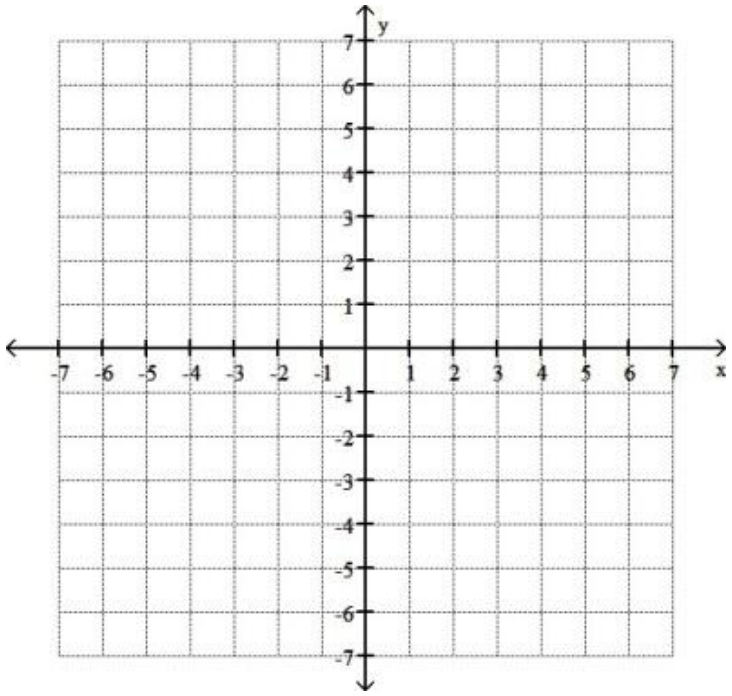
$$C = \{(-4,1),(-2,-1),(-2,0)\}$$



Tables and Graphs

$$A = \{(1,1), (-2,4), (3,-1)\}$$

x	y



Equations (or rule)

- A relation or function can be described using a rule that tells how to determine the dependent variable for a specific value of the independent variable.

Domains and Ranges of relations

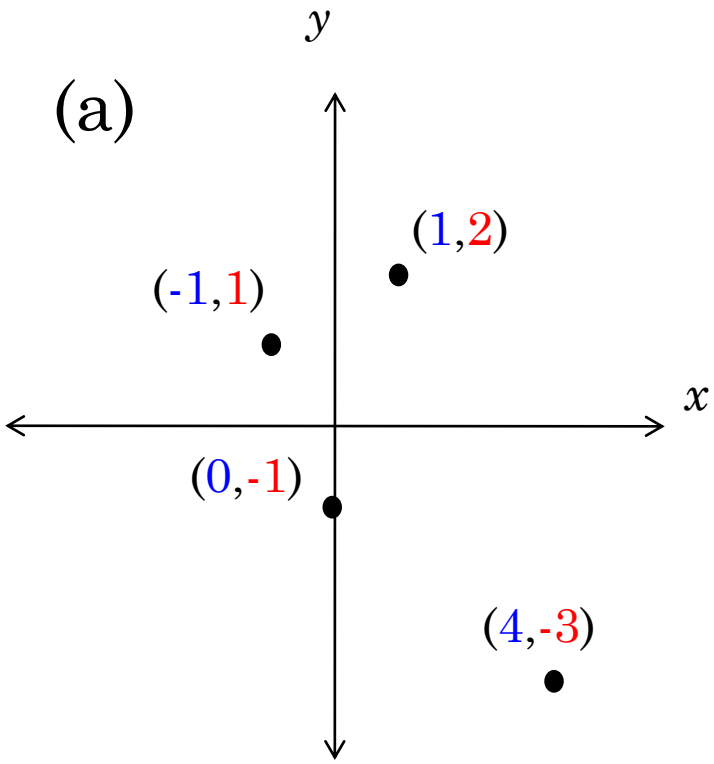
Domain and Range

Domains and Ranges of relations

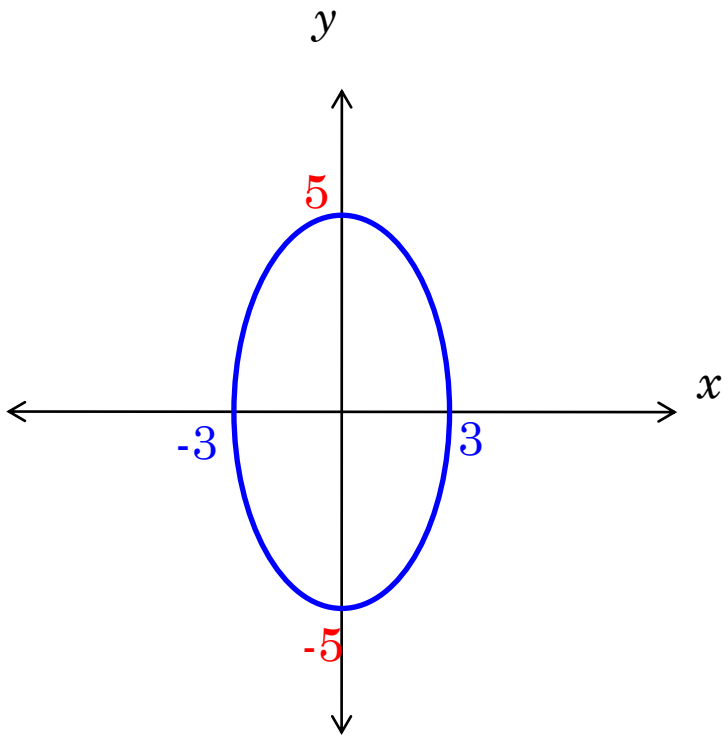
$$A = \{(1,2),(-2,4),(3,-1)\}$$

- Give the domain and range of each relation.

(a)

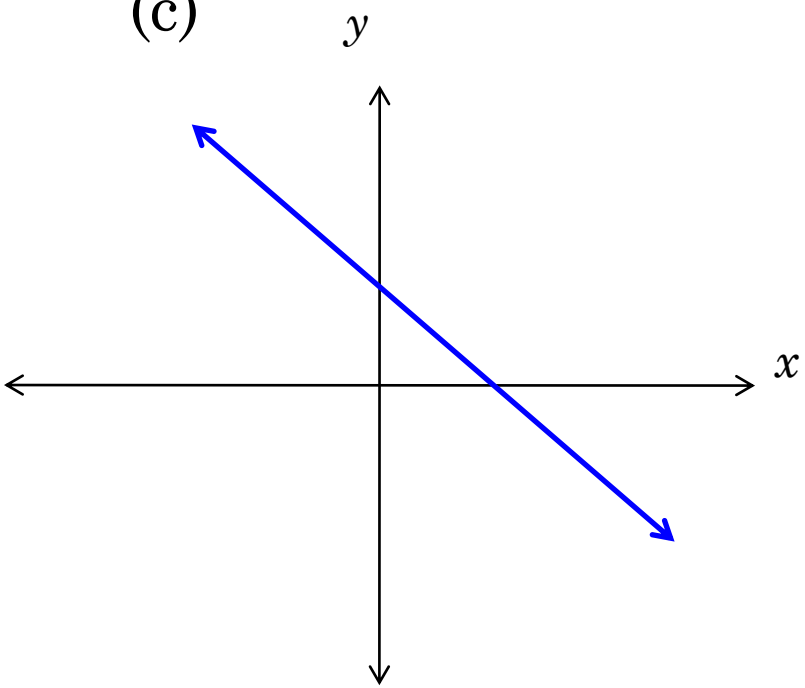


(b)

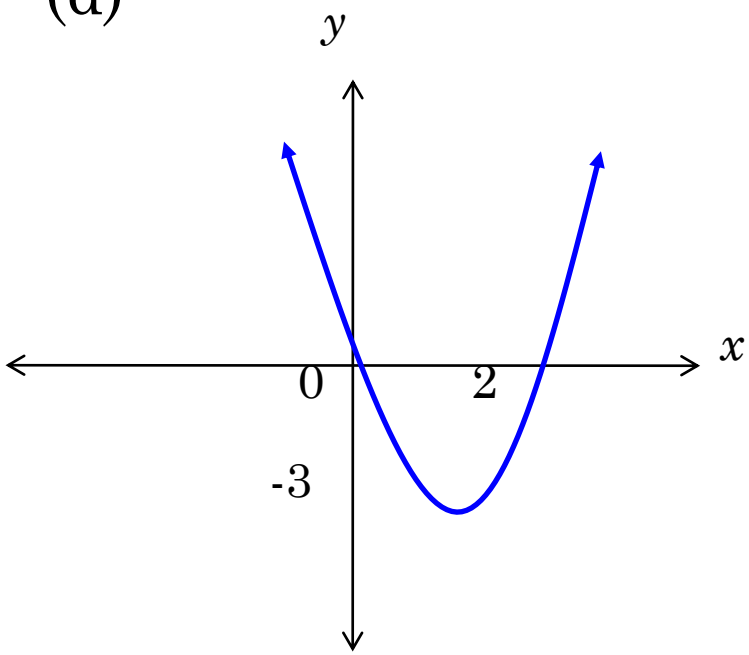


- Give the domain and range of each relation.

(c)

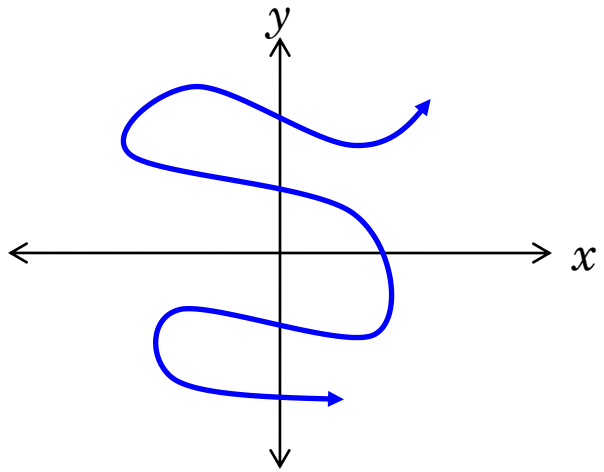


(d)

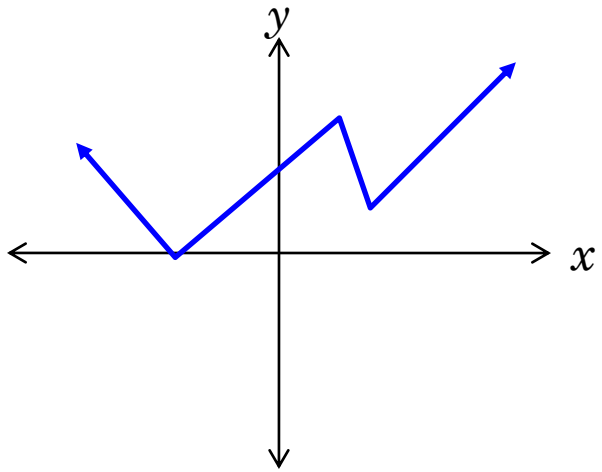


Identifying functions defined by graphs and equations

- Vertical Line Test – if every vertical line intersects the graph of a relation in no more than one point, then the relation is a function.



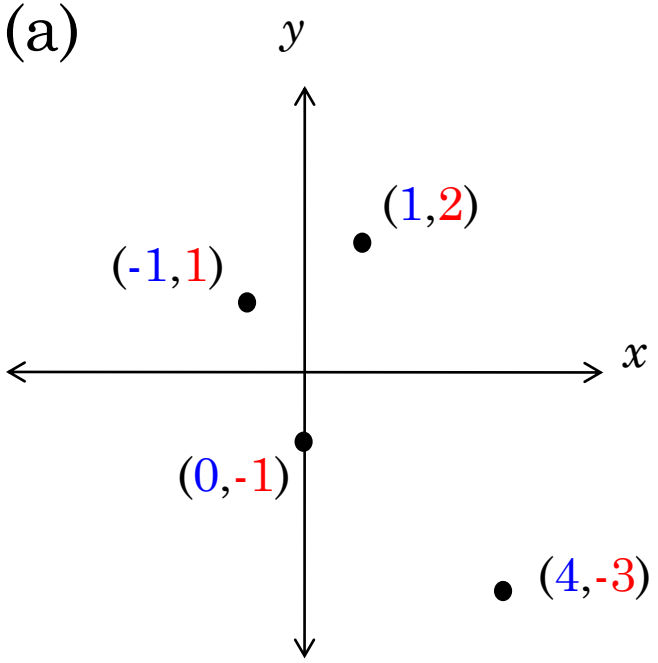
This is not the graph of a function. The line vertical line passes through more than one point.



This is the graph of a function. The vertical line will only ever pass through one point.

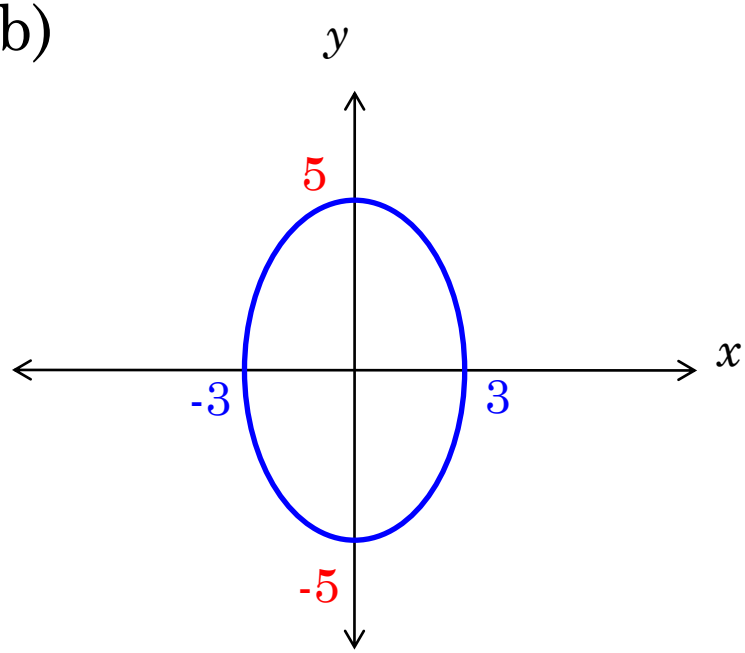
- Use the vertical line test to decide whether the relation graphed is a function.

(a)



This is the graph of a **function**. The vertical line will only ever pass through one point.

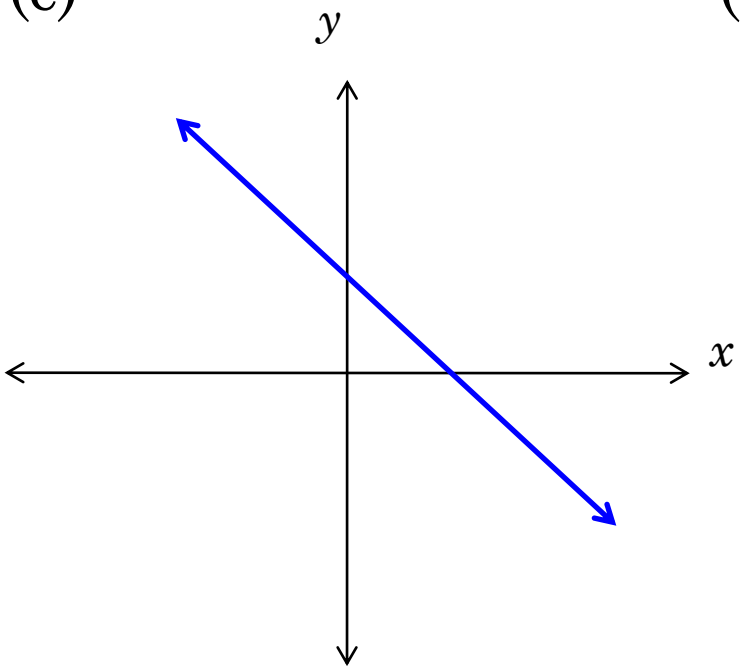
(b)



This is **not** the graph of a **function**. The line vertical line passes through more than one point.

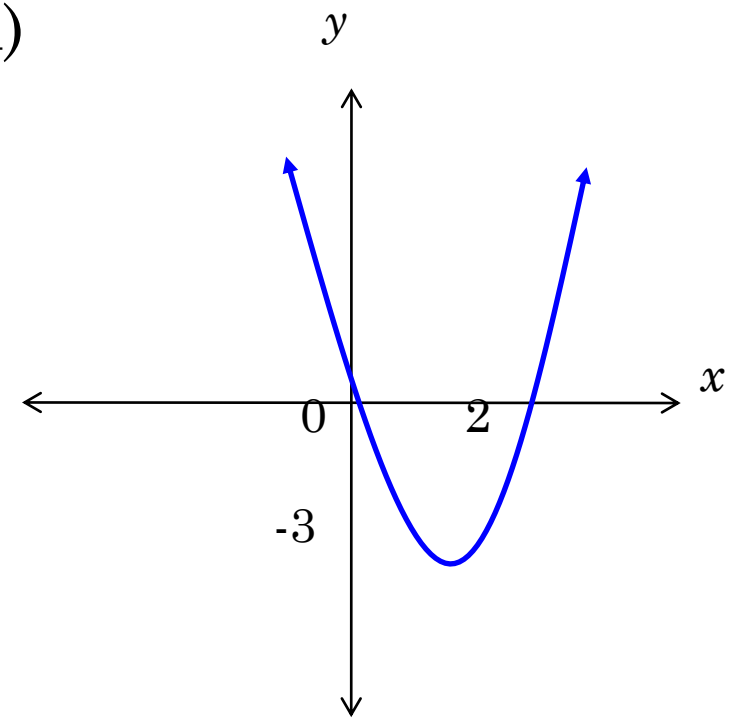
- Use the vertical line test to decide whether the relation graphed is a function.

(c)



This is the graph of a **function**. The vertical line will only ever pass through one point.

(d)



This is the graph of a **function**. The vertical line will only ever pass through one point.

Domain of Relations

Relations are often defined by equations. If a relation is defined by an equation keep the following guidelines in mind when finding domain.

- Exclude from the domain any values that make the denominator of a fraction equal to 0.

- Exclude from the domain any values that result in an even root of a negative number.

Identifying Functions from their Equations

- Decide whether each equation defines y as a function of x, and give the domain.

(a) $y = -2x + 7$

(b) $y = \sqrt{5x - 6}$

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(c) $y^4 = x$

(d) $y \geq 4x + 2$

Identifying Functions from their Equations

- Decide whether each equation defines y as a function of x, and give the domain.

(e) $y = \dfrac{6}{5+3x}$