3.1 Introduction to Fractions

Fractions tend to be one topic that many students dread. If this is you, we hope to change your opinion about fractions. If you take a look at what a fraction is, there is really nothing to worry about. A fraction is part of an integer. For example, if you order a large pizza to share with your roommates, and the pizza has twelve slices, each slice is considered to be $\frac{1}{12}$ of the pizza. If you eat four slices, you would be eating $\frac{4}{12}$ of the pizza or $\frac{1}{3}$ of the pizza. What would happen if you opened your pizza box and one of the slices was missing? This could be represented as $-\frac{1}{12}$, which means we are minus one of twelve slices.

The first thing you need to know about fractions, is the different parts of a fraction. There are three different parts of a fraction: numerator, denominator, and fraction bar. The **numerator** is the number or expression on the top. The **denominator** is the number or expression on the bottom. (To help you remember this, denominator starts with "d" for "down.") The line in the fraction is known as the **fraction bar**, which means to divide.

$$\frac{numerator}{denominator} \leftarrow \textit{fraction bar}$$

A fraction whose numerator is less than the denominator is called a **proper fraction**. The following are examples of proper fractions:

$$\frac{1}{2}, \quad -\frac{3}{4}, \quad \frac{7}{8}, \quad \text{and} \quad -\frac{5}{6}$$

A fraction whose numerator is greater than or equal to the denominator is called an **improper fraction**. The following are examples of improper fractions:

$$-\frac{11}{7}, \quad \frac{10}{10}, \quad -\frac{4}{2}, \quad \text{and} \quad \frac{3}{1}$$

One of the more common improper fractions you will see is an integer. An integer can be written as an improper fraction by placing a 1 in the denominator of the number. For example, the number 5 can be written:

$$5 = \frac{5}{1}$$

A very important concept to remember when working with fractions is the fact that the **denominator** of a fraction cannot equal **0**. Division by 0 is undefined.

$$\frac{5}{0} = undefined \quad \text{and} \quad \frac{-2}{0} = undefined$$

It is also important to note, when the numerator of a fraction is a zero, the value of the fraction is 0. Division into zero always equals 0.

$$\frac{0}{3}=0 \quad \text{and} \quad \frac{0}{-9}=0$$

Any improper fraction can take on another form. It can be written as a **mixed number**. A mixed number is made up of an integer and a proper fraction. The following are examples of mixed numbers:

$$1\frac{1}{3}, \quad 7\frac{6}{9}, \quad -3\frac{2}{5}, \quad \text{and} \quad 4\frac{1}{8}$$

It is important to note that in a mixed number, the integer and the fraction are added together. For example:

$$1\frac{1}{3}=1+\frac{1}{3} \qquad \text{and} \qquad -2\frac{1}{4}=-\left(2+\frac{1}{4}\right)$$

Regardless of how a fraction may be written, if it is **negative**, it remains **negative**. All negative fractions can be rewritten and (–1) multiplied by the fraction. A negative fraction can take on three different forms:

$$-\frac{1}{2}=\frac{-1}{2}=\frac{1}{-2}$$

The negative may be found in front of the fraction, in the numerator, or in the denominator. All three fractions shown above are equivalent.

Example 1

Identify the following as a proper fraction, improper fraction, or a mixed number.

a) $2\frac{1}{2}$ *mixed number* *There is a whole number and a fraction.*

b) $-\frac{5}{8}$ *proper fraction* *The numerator is smaller than the denominator.*

c) $\frac{12}{7}$ *improper fraction* *The numerator is larger than the denominator.*

d) $\frac{3}{5}$ *proper fraction* *The numerator is smaller than the denominator.*

e) $\frac{1}{5}$ *proper fraction* *The numerator is* *__smaller__* *than the denominator.*

f) $9\frac{6}{11}$ *mixed number* *There is a whole number and a fraction.*

g) $-\frac{13}{13}$ *improper fraction* *The numerator is* *__equal__* *to the denominator.*

h) $-3\frac{4}{5}$ *mixed number* *There is a whole number and a fraction.*

Now take a closer look at improper fractions and mixed numbers. An improper fraction can be rewritten as a mixed number and a mixed number can be rewritten as an improper fraction.

CHANGING AN IMPROPER FRACTION INTO A MIXED NUMBER

To change an improper fraction into a mixed number you can write the fraction long-handed. This is done by rewriting the numerator of the fraction as a sum of 1's, over the denominator. Try this with the following fraction:

$$\frac{13}{5} = \frac{1+1+1+1+1+1+1+1+1+1+1+1+1}{5}$$

After rewriting the fraction, look at the denominator. The denominator tells you how many 1's it takes to make a whole number. In our example we need five 1's for each integer. Circle each group of five.

$$\frac{13}{5} = \frac{\boxed{1+1+1+1+1}+\boxed{1+1+1+1+1}+1+1+1}{5}$$

We have two groups circled. Therefore, the integer part of our mixed number is 2. We have three 1's remaining, so the fraction part of our mixed number has a numerator of 3 and our denominator stay the same, 5. Giving us the mixed number:

$$2\frac{3}{5}$$

Example 2

Change the following improper fractions into mixed numbers doing it long-handed.

a)
$$\frac{11}{4}=\frac{1+1+1+1+1+1+1+1+1+1+1}{4}$$
$$=\frac{\boxed{1+1+1+1}+\boxed{1+1+1+1}+1+1+1}{4}$$
$$=2\frac{3}{4}$$

Our denominator is 4, so we are looking for groups of 4.

b)
$$-\frac{7}{5}=-\frac{1+1+1+1+1+1+1}{5}$$
$$=-\frac{\boxed{1+1+1+1+1}+1+1}{5}$$
$$=-1\frac{2}{5}$$

Our denominator is 5, so we are looking for groups of 5.

The fraction is negative and will remain negative.

Now that you can change an improper fraction into a mixed number, examine the following fraction:

$$\frac{12}{4}=\frac{1+1+1+1+1+1+1+1+1+1+1+1}{4}$$
$$=\frac{\boxed{1+1+1+1}+\boxed{1+1+1+1}+\boxed{1+1+1+1}}{4}$$

Notice, after writing it out long-handed, there are no 1's remaining. When this happens, our improper fraction becomes an integer. In the example above, we can use the fraction bar, which means divide, to divide 12 by 4 giving us a quotient of 3. This brings us to another method we can use to change an improper fraction to a mixed number, division.

To use division, divide the numerator by the denominator. The quotient you get is the integer part of your mixed number. The remainder is the numerator of the fraction part, over the same denominator. Look at the following example:

$\frac{13}{3}$ Use long division to divide 13 by 3:

$$\begin{array}{r} 4 \leftarrow \textit{quotient} \\ 3\overline{)13} \\ \underline{-12} \\ 1 \leftarrow \textit{remainder} \end{array}$$

we then have the mixed number: $4\frac{1}{3}$

Example 3

Change the following improper fractions into mixed numbers using division.

a) $\dfrac{5}{3}$

$$\begin{array}{r} 1 \leftarrow \textit{quotient} \\ 3\overline{)\,5} \\ \underline{-3} \\ 2 \leftarrow \textit{remainder} \end{array}$$

$= 1\dfrac{2}{3}$

b) $\dfrac{64}{4}$

$$\begin{array}{r} 16 \leftarrow \textit{quotient} \\ 4\overline{)\,64} \\ \underline{-4} \\ 24 \\ \underline{-24} \\ 0 \leftarrow \textit{remainder} \end{array}$$

$= 16$

c) $-\dfrac{13}{9}$

$$\begin{array}{r} 1 \leftarrow \textit{quotient} \\ 9\overline{)13} \\ \underline{-9} \\ 4 \leftarrow \textit{remainder} \end{array}$$

$= -1\dfrac{4}{9}$

The fraction is negative and will remain negative.

CHANGING A MIXED NUMBER INTO AN IMPROPER FRACTION

When we changed an improper fraction into a mixed number, we used division. To change a mixed number into an improper fraction, we will use the opposite operation, multiplication.

To change an improper fraction into a mixed number, multiply the denominator by the integer. Take that product and add the numerator. This result is the numerator of the improper fraction. The denominator stays the same. The following illustrates this concept:

$$9\frac{1}{2} = 9\frac{1}{2} = \frac{2\cdot 9+1}{2} = \frac{18+1}{2} = \frac{19}{2}$$

Example 4

Change the following mixed numbers into improper fractions.

a) $4\dfrac{7}{10} = 4\dfrac{7}{10} = \dfrac{10\cdot 4+7}{10} = \dfrac{40+7}{10} = \dfrac{47}{10}$

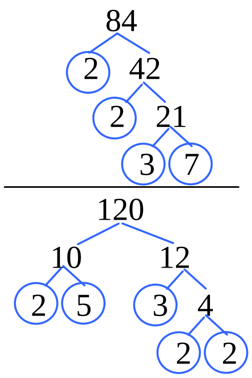
b) $-5\dfrac{1}{6} = -\left(5\dfrac{1}{6}\right) = -\left(\dfrac{6\cdot 5+1}{6}\right) = -\left(\dfrac{30+1}{6}\right) = -\dfrac{31}{6}$

The fraction is negative and will remain negative.
Keep the negative sign in the front.

REDUCING FRACTIONS

Have you ever heard of "**reducing**" or "**simplifying fractions**"? What exactly does this mean? Reducing a fraction means eliminating any factors the numerator and denominator have in common. Reducing a fraction is also called "**writing it in lowest terms**". Reducing can be done with the use of a factor tree.

Use a factor tree to rewrite the numerator as a product of prime numbers. Do the same thing for the denominator. For example:



$$\frac{84}{120} = \frac{2\cdot 2\cdot 3\cdot 7}{2\cdot 2\cdot 2\cdot 3\cdot 5}$$

For the next step, we need to remember, when you multiply by the number one, it does not change the value of your number. **1 takes the form of any expression divided by itself.** For example:

$$\frac{2}{2}=1 \qquad \frac{5}{5}=1 \qquad \frac{10}{10}=1 \qquad \frac{x}{x}=1,\ \text{and so on.}$$

Now let's look for the ones in our fraction, $\frac{84}{120}$.

$$\frac{84}{120}=\frac{2\cdot 2\cdot 3\cdot 7}{2\cdot 2\cdot 2\cdot 3\cdot 5}=\frac{2}{2}\cdot\frac{2}{2}\cdot\frac{3}{3}\cdot\frac{7}{2\cdot 5}$$

$$= 1\cdot 1\cdot 1\cdot \frac{7}{10} = \frac{7}{10}$$

We can shorten the process of reducing by using what we refer to as "**the long bar"** method. This method is done by writing a long bar. We then place the prime factors of the numerator on the top and the prime factors of the denominator on the bottom. Make sure that everything is a product. We then find the ones.

$$\frac{60}{54}=\frac{2\cdot 2\cdot 3\cdot 5}{2\cdot 3\cdot 3\cdot 3}$$

$$=\frac{\overset{1}{\cancel{2}}\cdot 2\cdot \overset{1}{\cancel{3}}\cdot 5}{\underset{1}{\cancel{2}}\cdot \underset{1}{\cancel{3}}\cdot 3\cdot 3}$$

$$=\frac{2\cdot 5}{3\cdot 3}=\frac{10}{9}$$

What happens once we have found all the ones in the long bar method and there are no numbers remaining in the numerator or denominator? Look at the following examples:

$$\frac{15}{30}=\frac{3\cdot 5}{2\cdot 3\cdot 5}=\frac{?}{2} \qquad \textit{and} \qquad \frac{350}{35}=\frac{2\cdot 5\cdot 5\cdot 7}{5\cdot 7}=\frac{2\cdot 5}{?}=\frac{10}{?}$$

You may think that the fraction goes to zero because there is nothing left. (Recall, that you can't divide by zero.) This is not the case. When we have found our ones, as in $\frac{5}{5}$, the number does not disappear. The $\frac{5}{5}$ is actually replaced with it's value, **1**. Therefore, if there are no remaining numbers, we are left with a 1 in that place. So,

$$\frac{15}{30}=\frac{\overset{1}{\cancel{3}}\cdot \overset{1}{\cancel{5}}}{2\cdot \underset{1}{\cancel{3}}\cdot \underset{1}{\cancel{5}}}=\frac{1}{2} \qquad \textit{and} \qquad \frac{350}{35}=\frac{2\cdot \overset{1}{\cancel{5}}\cdot 5\cdot \overset{1}{\cancel{7}}}{\underset{1}{\cancel{5}}\cdot \underset{1}{\cancel{7}}}=\frac{2\cdot 5}{1}=\frac{10}{1}=10$$

This should be written as an integer, 10.

Example 5

Reduce or simplify the following fractions to lowest terms.

$$\frac{12}{16} = \frac{2\cdot 2\cdot 3}{2\cdot 2\cdot 2\cdot 2} = \frac{\overset{1}{\cancel{2}}\cdot \overset{1}{\cancel{2}}\cdot 3}{\underset{1}{\cancel{2}}\cdot \underset{1}{\cancel{2}}\cdot 2\cdot 2}$$

$$= \frac{3}{2\cdot 2}$$

$$= \frac{3}{4}$$

Example 6

Reduce or simplify the following fractions to lowest terms.

$$\frac{8}{15} = \frac{2\cdot 2\cdot 2}{3\cdot 5}$$

There are no 1's. The fraction is already in lowest terms.

$$= \frac{8}{15}$$

Example 7

Reduce or simplify the following fractions to lowest terms.

$$-\frac{96}{124} = \frac{2\cdot 2\cdot 2\cdot 2\cdot 2\cdot 3}{2\cdot 2\cdot 31} = -\frac{\overset{1}{\cancel{2}}\cdot \overset{1}{\cancel{2}}\cdot 2\cdot 2\cdot 2\cdot 3}{\underset{1}{\cancel{2}}\cdot \underset{1}{\cancel{2}}\cdot 31}$$

$$= -\frac{2\cdot 2\cdot 2\cdot 3}{31}$$

$$= -\frac{24}{31}$$

The fraction is negative and will remain negative.

The long bar method of reducing works with numbers as well as with variables. When a variable occurs in a fraction, write it in expanded form. Look for the ones in your fraction and simplify in the same manner you do with numbers.

Note: Fractions are undefined if the denominator is equal to zero. For this reason, we will assume that all variables in this chapter are not equal to zero.

Example 8

Reduce or simplify the following fractions to lowest terms.

$$\frac{15x^3}{25x} = \frac{3\cdot 5\cdot x\cdot x\cdot x}{5\cdot 5\cdot x} = \frac{3\cdot \overset{1}{\cancel{5}}\cdot \overset{1}{\cancel{x}}\cdot x\cdot x}{\underset{1}{\cancel{5}}\cdot 5\cdot \underset{1}{\cancel{x}}}$$

$$= \frac{3\cdot x\cdot x}{5}$$

$$= \frac{3x^2}{5}$$

Example 9

Reduce or simplify the following fractions to lowest terms.

$$\frac{36x^2y^4}{9x^5y^2} = \frac{2\cdot 2\cdot 3\cdot 3\cdot x\cdot x\cdot y\cdot y\cdot y\cdot y}{3\cdot 3\cdot x\cdot x\cdot x\cdot x\cdot x\cdot y\cdot y}$$

$$= \frac{2\cdot 2\cdot \overset{1}{\cancel{3}}\cdot \overset{1}{\cancel{3}}\cdot \overset{1}{\cancel{x}}\cdot \overset{1}{\cancel{x}}\cdot \overset{1}{\cancel{y}}\cdot \overset{1}{\cancel{y}}\cdot y\cdot y}{\underset{1}{\cancel{3}}\cdot \underset{1}{\cancel{3}}\cdot \underset{1}{\cancel{x}}\cdot \underset{1}{\cancel{x}}\cdot x\cdot x\cdot x\cdot \underset{1}{\cancel{y}}\cdot \underset{1}{\cancel{y}}}$$

$$= \frac{2\cdot 2\cdot y\cdot y}{x\cdot x\cdot x}$$

$$= \frac{4y^2}{x^3}$$

3.1 EXERCISES

In 1-5, identify the following as a proper fraction, improper fraction, or mixed number.

1. $-\frac{3}{5}$

2. $113\frac{1}{3}$

3. $-\frac{12}{5}$

4. $\frac{17}{17}$

5. $\frac{4}{9}$

In 6-10, change the following improper fractions into mixed numbers using division.

6. $\frac{8}{3}$

7. $-\frac{13}{4}$

8. $\frac{25}{5}$

9. $\frac{35}{6}$

10. $-\frac{100}{9}$

In 11-15, change the following mixed numbers into improper fractions.

11. $-7\frac{2}{3}$

12. $3\frac{5}{6}$

13. $2\frac{11}{12}$

14. $-8\frac{3}{5}$

15. $15\frac{1}{4}$

In 16-23, reduce or simplify the following fractions to lowest terms.

16. $\frac{16}{20}$

17. $-\frac{60}{42}$

18. $\frac{x^5}{x^7}$

19. $\frac{11}{13}$

20. $-\frac{21a^6}{63a^3}$

21. $\frac{100}{28}$

22. $-\frac{15x^9y^4}{125x^6y^7}$

23. $\frac{4a^3b^2c^4d^0}{16ab^2c^3}$

Applications.

24. In a math class there are sixteen females. The total number of students in the class is forty. Write a fraction of the total number of students that are female. Reduce if possible.

25. Tony's girlfriend has made him a banana cream pie. Tony cut the pie into eight pieces and ate three of the pieces. Represent the number of pieces that were eaten of the total pieces as a fraction. Reduce if possible.

3.2 Multiplication of Fractions

Your English professor asks you how many hours per week you spend outside of class reading. You spend $\frac{1}{2}$ hour per day reading. How many hours per week will you tell your professor you spend reading?

There are seven days in one week so we take $\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}$, which gives us seven halves and can be written as $\frac{7}{2}$. In this example, there is a shorter way to simplify the problem. It can be done with the use of multiplication. There are 7 days and you read $\frac{1}{2}$ hour each day. Multiply 7 by $\frac{1}{2}$. To do this, write 7 as an improper fraction, $\frac{7}{1}$, then multiply. Multiply the numerators together, then, multiply the denominators together.

$$\frac{7}{1}\cdot\frac{1}{2} = \frac{7\cdot 1}{1\cdot 2} = \frac{7}{2}$$

MULTIPLICATION OF FRACTIONS

To multiply fractions, multiply the numerators together, then, multiply the denominators together. (*b* and *d* are not equal to zero.)

$$\frac{a}{b}\cdot\frac{c}{d} = \frac{a\cdot c}{b\cdot d}$$

When multiplying fractions, we need to make sure our answer is in lowest terms. After we find the product, we will reduce the fraction using "the long bar." (Remember to use a factor tree to find the prime numbers.)

Example 1 **Find the product. Reduce to lowest terms.** $\frac{8}{15}\cdot\frac{25}{28}$

$$\frac{8}{15}\cdot\frac{25}{28} = \frac{8\cdot 25}{15\cdot 28} = \frac{200}{420}$$

$$\frac{200}{420} = \frac{\cancelto{1}{2}\cdot\cancelto{1}{2}\cdot 2\cdot\cancelto{1}{5}\cdot 5}{\cancelto{1}{2}\cdot\cancelto{1}{2}\cdot 3\cdot\cancelto{1}{5}\cdot 7} = \frac{2\cdot 5}{3\cdot 7}$$

$$= \frac{10}{21}$$

Example 2

Find the product. Reduce to lowest terms. $\dfrac{20}{21}\cdot\dfrac{7}{10}$

$$\frac{20}{21}\cdot\frac{7}{10} = \frac{20\cdot 7}{21\cdot 10} = \frac{140}{210}$$

$$\frac{140}{210} = \frac{\cancel{2}^{1}\cdot 2\cdot \cancel{5}^{1}\cdot \cancel{7}^{1}}{\cancel{2}_{1}\cdot 3\cdot \cancel{5}_{1}\cdot \cancel{7}_{1}}$$

$$= \frac{2}{3}$$

To shorten this process, we can use the "long bar" before we find the product.

$$\frac{20}{21}\cdot\frac{7}{10} = \frac{2\cdot 2\cdot 5\ \cdot\ 7}{3\cdot 7\ \cdot\ 2\cdot 5}$$

$$= \frac{\cancel{2}^{1}\cdot 2\cdot \cancel{5}^{1}\cdot \cancel{7}^{1}}{3\cdot \cancel{7}_{1}\cdot \cancel{2}_{1}\cdot \cancel{5}_{1}}$$

$$= \frac{2}{3}$$

In multiplication of fractions, there is another way to reduce the fraction. This is done before we find the product. This is commonly known as "**cross-canceling**" and can **only** be done with multiplication of fractions. To cross-cancel, look for a common factor in the numerator and denominator of either fraction. Then divide each number by that common factor. Find the product of the remaining numbers. This concept is illustrated in the example below.

$$\frac{20^{\div 10}}{21}\cdot\frac{7}{10_{\div 10}} = \frac{2}{21}\cdot\frac{7}{1}$$ *The 20 & 10 share a factor of 10. Divide each by 10.*

$$= \frac{2}{21_{\div 7}}\cdot\frac{7^{\div 7}}{1}$$ *The 21 & 7 share a factor of 7. Divide each by 7.*

$$= \frac{2}{3}\cdot\frac{1}{1}$$ *Now, find the product.*

$$= \frac{2}{3}$$

MULTIPLYING MIXED NUMBERS

To multiply mixed numbers, you must first change the mixed numbers into improper fractions. Then multiply using any of the methods we have discussed previously.

$$2\frac{2}{3}\cdot 5\frac{3}{4}=\frac{8}{3}\cdot\frac{23}{4}=$$

"long bar" $$\frac{8}{3}\cdot\frac{23}{4}=\frac{\cancelto{1}{2}\cdot\cancelto{1}{2}\cdot 2\cdot 23}{3\cdot\cancelto{1}{2}\cdot\cancelto{1}{2}}=\frac{46}{3}$$

"cross canceling" $$\frac{\cancelto{2}{8}}{3}\cdot\frac{23}{\cancelto{1}{4}}=\frac{46}{3}$$

After looking at the different ways to multiply and reduce a fraction, it is important that you choose which method works best for you. It is also important to remember the rules for multiplication of signed numbers.

like signs	positive $\times\!/\!\div$ positive = positive negative $\times\!/\!\div$ negative = positive
unlike signs	positive $\times\!/\!\div$ negative = negative negative $\times\!/\!\div$ posititve = negative

All of the examples we have shown up to this point have us finding the product of two fractions. The methods of multiplication do not depend on how many fractions we find the product of. We can use these methods to find the product of several fractions at one time.

Example 3 **Find the product. Reduce to lowest terms.** $\frac{3}{4}\cdot\frac{2}{9}\cdot\frac{3}{5}$

$$\frac{3}{4}\cdot\frac{2}{9}\cdot\frac{3}{5}=$$

"long bar" $$\frac{3}{4}\cdot\frac{2}{9}\cdot\frac{3}{5}=\frac{\cancelto{1}{3}\cdot\cancelto{1}{2}\cdot\cancelto{1}{3}}{\cancelto{1}{2}\cdot 2\cdot\cancelto{1}{3}\cdot\cancelto{1}{3}\cdot 5}=\frac{1}{10}$$

"cross canceling" $$\frac{\cancelto{1}{3}}{\cancelto{2}{4}}\cdot\frac{\cancelto{1}{2}}{\cancelto{1}{\cancelto{3}{9}}}\cdot\frac{\cancelto{1}{3}}{5}=\frac{1}{10}$$

When working with variables, we suggest using the “long bar” method. In Chapter 9, we will discuss another method of simplifying with fractions.

Example 4

Find the product. Reduce to lowest terms. $\dfrac{x^2y}{z}\cdot\dfrac{yz^3}{x^5}$

$$\begin{aligned}\frac{x^2y}{z}\cdot\frac{yz^3}{x^5} &= \frac{\overset{1}{\cancel{x}}\cdot\overset{1}{\cancel{x}}\cdot y\cdot y\cdot\overset{1}{\cancel{z}}\cdot z\cdot z}{\underset{1}{\cancel{z}}\cdot\underset{1}{\cancel{x}}\cdot\underset{1}{\cancel{x}}\cdot x\cdot x\cdot x}\\ &= \frac{y\cdot y\cdot z\cdot z}{x\cdot x\cdot x}\\ &= \frac{y^2z^2}{x^3}\end{aligned}$$

Example 5

Find the product. Reduce to lowest terms. $\left(-\dfrac{1}{5}\right)^3$

$$\begin{aligned}\left(-\frac{1}{5}\right)^3 &= \left(-\frac{1}{5}\right)\cdot\left(-\frac{1}{5}\right)\cdot\left(-\frac{1}{5}\right)\\ &= \frac{-1\cdot-1\cdot-1}{5\cdot5\cdot5}\\ &= -\frac{1}{125}\end{aligned}$$

The product of three negatives is a negative. Your final answer will be negative.

Example 6

Find the product. Reduce to lowest terms. $7\dfrac{1}{7}\cdot 8\cdot 6\dfrac{3}{10}$

$$7\frac{1}{7}\cdot 8\cdot 6\frac{3}{10} = \frac{50}{7}\cdot\frac{8}{1}\cdot\frac{63}{10}$$

Start by rewriting the mixed numbers as improper fractions.

$\dfrac{50}{7}\cdot\dfrac{8}{1}\cdot\dfrac{63}{10}=$

"long bar"

$$\frac{50}{7}\cdot\frac{8}{1}\cdot\frac{63}{10}=\frac{\overset{1}{\cancel{2}}\cdot\overset{1}{\cancel{5}}\cdot5\cdot2\cdot2\cdot2\cdot3\cdot3\,\overset{1}{\cancel{7}}}{\underset{1}{\cancel{7}}\cdot1\cdot\underset{1}{\cancel{2}}\;\underset{1}{\cancel{5}}}=\frac{360}{1}=360$$

"cross canceling"

$$\frac{\overset{5}{\cancel{50}}}{\underset{1}{\cancel{7}}}\cdot\frac{8}{1}\cdot\frac{\overset{9}{\cancel{63}}}{\underset{1}{\cancel{10}}}=\frac{360}{1}=360$$

Example 7

Find the product. Reduce to lowest terms. $-\frac{15x^2y}{32x}\cdot-\frac{8xy}{5yz}$

$$-\frac{15x^2y}{32x}\cdot-\frac{8xy}{5yz} = \frac{3\cdot\cancel{5}^{1}\cdot\cancel{x}^{1}\cdot x\cdot y\quad\cdot\cancel{2}^{1}\cdot\cancel{2}^{1}\cdot\cancel{2}^{1}\cdot x\cdot\cancel{y}^{1}}{\cancel{2}_{1}\cdot\cancel{2}_{1}\cdot\cancel{2}_{1}\cdot 2\cdot 2\quad\cdot\cancel{5}_{1}\cdot\cancel{y}_{1}\cdot z}$$

A negative multiplied by a negative, results in a positive answer.

$$= \frac{3\cdot x\cdot x\cdot y}{2\cdot 2\cdot z}$$

$$= \frac{3x^2y}{4z}$$

Example 8

Find the product. Reduce to lowest terms. $\left(\frac{3}{4}\right)^2\cdot\left(-\frac{2}{3}\right)^3$

$$\left(\frac{3}{4}\right)^2\cdot\left(-\frac{2}{3}\right)^3 = \left(\frac{3}{4}\right)\left(\frac{3}{4}\right)\cdot\left(-\frac{2}{3}\right)\left(-\frac{2}{3}\right)\left(-\frac{2}{3}\right)$$

$$= \left(\frac{\cancel{3}^{1}}{\cancel{2}_{1}\cdot\cancel{2}_{1}}\right)\left(\frac{\cancel{3}^{1}}{\cancel{2}_{1}\cdot 2}\right)\cdot\left(-\frac{\cancel{2}^{1}}{\cancel{3}_{1}}\right)\left(-\frac{\cancel{2}^{1}}{\cancel{3}_{1}}\right)\left(-\frac{\cancel{2}^{1}}{3}\right)$$

The product of three negatives is a negative. Your final answer will be negative.

$$= -\frac{1}{2\cdot 3}$$

$$= -\frac{1}{6}$$

Example 9

Multiply $\frac{3}{4}$ and $\frac{8}{11}$. Reduce to lowest terms.

$\frac{3}{4}\cdot\frac{8}{11} =$

"long bar" $\quad \frac{3}{4}\cdot\frac{8}{11} = \frac{3\cdot\cancel{2}^{1}\cdot\cancel{2}^{1}\cdot 2}{\cancel{2}_{1}\cdot\cancel{2}_{1}\cdot 11} = \frac{6}{11}$

"cross canceling" $\quad \frac{3}{\cancel{4}_{1}}\cdot\frac{\cancel{8}^{2}}{11} = \frac{6}{11}$

Example 10

Find the product of $2\frac{7}{8}, 1\frac{11}{13}$, **and** $4\frac{1}{2}$**. Reduce to lowest terms.**

$$2\frac{7}{8}\cdot 1\frac{11}{13}\cdot 4\frac{1}{2} = \frac{23}{8}\cdot\frac{24}{13}\cdot\frac{9}{2}$$

Start by rewriting the mixed numbers as improper fractions.

"long bar"

$$\frac{23}{8}\cdot\frac{24}{13}\cdot\frac{9}{2} = \frac{23\cdot\overset{1}{\cancel{2}}\cdot\overset{1}{\cancel{2}}\cdot\overset{1}{\cancel{2}}\cdot 3\cdot 3\cdot 3}{\underset{1}{\cancel{2}}\cdot\underset{1}{\cancel{2}}\cdot\underset{1}{\cancel{2}}\cdot 13\cdot 2} = \frac{621}{26}$$

$$2\frac{23}{8}\cdot\frac{24}{13}\cdot\frac{9}{2} =$$

"cross canceling"

$$\frac{23}{\underset{1}{\cancel{8}}}\cdot\frac{\overset{3}{\cancel{24}}}{13}\cdot\frac{9}{2} = \frac{621}{26}$$

Example 11

***Application*. In a college history class, two-elevenths of the students did <u>not</u> pass the class. If there are thirty-three students in the class, how many students did <u>not</u> pass the class? How many students passed the class?**

The word "of" means to multiply, in order to solve this problem, we must multiply two-elevenths by thirty-three.

$$\frac{2}{11}\cdot 33 = \frac{2}{11}\cdot\frac{33}{1}$$

Start by rewriting the whole number as an improper fraction.

"long bar"

$$\frac{2}{11}\cdot\frac{33}{1} = \frac{2\cdot 3\cdot\overset{1}{\cancel{11}}}{\underset{1}{\cancel{11}}\cdot 1} = \frac{6}{1} = 6$$

$$\frac{2}{11}\cdot\frac{33}{1} =$$

"cross canceling"

$$\frac{2}{\underset{1}{\cancel{11}}}\cdot\frac{\overset{3}{\cancel{33}}}{1} = \frac{6}{1} = 6$$

The number of students that did <u>not</u> pass the class is **6**.

The number of students that passed the class is 33 – 6 = **27**.

3.2 EXERCISES

In 1-18, multiply the following fractions. Reduce all answers to lowest terms.

1. $\frac{2}{3} \cdot \frac{7}{5}$

2. $4 \cdot \frac{5}{2}$

3. $-\frac{6}{7} \cdot \frac{2}{5}$

4. $\frac{7}{8} \cdot \frac{15}{16}$

5. $\frac{14}{45} \cdot \frac{-3}{56}$

6. $10 \cdot \frac{1}{100} \cdot \frac{1}{10}$

7. $\frac{x}{y} \cdot \frac{y^2}{x}$

8. $\frac{3}{5} \cdot \frac{2}{9}$

9. $\frac{ab}{c} \cdot \frac{c^2}{b^2}$

10. $\frac{64x^2}{9y^2} \cdot \frac{72y}{16x}$

11. $\frac{-55}{19} \cdot \frac{-38}{11}$

12. $11\frac{5}{6} \cdot 3\frac{1}{2}$

13. $\frac{5}{16} \cdot 84$

14. $-1\frac{2}{3} \cdot 3\frac{3}{5}$

15. $\frac{1}{2} \cdot \frac{5}{6} \cdot \frac{8}{10}$

16. $\left(\frac{4}{3}\right)^3$

17. $\left(1\frac{1}{4}\right)^2$

18. $\frac{7}{8} \cdot \frac{2}{5} \cdot 6 \cdot \frac{15}{7}$

In 19-22, perform the indicated operation. Reduce all answers to lowest terms.

19. Find the product of $\frac{10}{13}$ and $\frac{3}{5}$.

20. Find the product of $10\frac{2}{3}$ and $8\frac{1}{2}$.

21. Find $\frac{1}{2}$ of $\frac{10}{3}$.

22. Find $\frac{3}{5}$ of $\frac{1}{3}$ of 15.

Applications.

23. A cook needs to triple a recipe that uses three and one-half cups of sugar. How much sugar will be needed?

24. A truck can carry seven-eighths ton of gravel. How many tons of gravel are in five loads?

25. In a history class five-sixths of the students passed. If there are thirty-six students in the class, how many students passed the class? How many students did not pass the class?

26. Hanna addresses three-eights of a box of wedding invitations each day. At this rate, how many boxes will she complete in four and two-thirds days?

3.3 Reciprocals & Division of Fractions

RECIPROCALS

When dividing fractions, we use what is known as a reciprocal. **Reciprocals** are two numbers that have a product of 1. The reciprocal of a fraction is written by flipping or inverting the fraction. For instance, the reciprocal of $\frac{3}{8}$ is $\frac{8}{3}$, the reciprocal of $\frac{-7}{8}$ is $\frac{-8}{7}$, and the reciprocal of $\frac{1}{4}$ is $\frac{4}{1}$, which can also be written as 4. When a fraction is negative, it's reciprocal is also negative. When a fraction is positive, it's reciprocal is also positive.

To find the reciprocal of a mixed number or a whole number, you must first make it an improper fraction and then invert it. For example:

$$2\frac{1}{5} = 2\frac{1}{5} = \frac{11}{5} \text{ so the reciprocal of } 2\frac{1}{5} \text{ is } \frac{5}{11}.$$

In order for two numbers to be reciprocals, their **product must equal 1.**

$\frac{4}{5}$ and $\frac{5}{4}$	are reciprocals because	$\frac{4}{5}\cdot\frac{5}{4}=\frac{20}{20}=1$
$-\frac{7}{4}$ and $-\frac{4}{7}$	are reciprocals because	$-\frac{7}{4}\cdot-\frac{4}{7}=\frac{28}{28}=1$
$\frac{1}{5}$ and 5	are reciprocals because	$\frac{1}{5}\cdot 5=\frac{1}{5}\cdot\frac{5}{1}=\frac{5}{5}=1$
$-8\frac{1}{6}$ and $-\frac{6}{49}$	are reciprocals because	$-8\frac{1}{6}\cdot-\frac{6}{49}=-\frac{49}{6}\cdot-\frac{6}{49}=\frac{294}{294}=1$
$\frac{x}{2}$ and $\frac{2}{x}$	are reciprocals because	$\frac{x}{2}\cdot\frac{2}{x}=\frac{2x}{2x}=1$

Zero does not have a reciprocal because you cannot multiply by zero and get a product equal to one. For example:

$\frac{0}{5}$ and $\frac{5}{0}$	are **not** reciprocals because	$\frac{0}{5}\cdot\frac{5}{0}\neq 1$

Recall, fractions are undefined if the denominator is equal to zero. For this reason, we will assume that all variables in this chapter are not equal to zero.

Example 1

Write the reciprocals of the following fractions.

	Fraction			Reciprocal
a)	$\frac{4}{5}$			$\frac{5}{4}$
b)	$\frac{3}{x^2}$			$\frac{x^2}{3}$
c)	-8	*Make the whole number into an improper fraction before inverting it.*	$-8 = -\frac{8}{1}$	$-\frac{1}{8}$
d)	$\frac{1}{7}$			$\frac{7}{1} = 7$
e)	$-4\frac{1}{8}$	*Make the mixed number into an improper fraction before inverting it.*	$-4\frac{1}{8} = -\frac{33}{8}$	$-\frac{8}{33}$

Division Of Fractions

Nicole has $\frac{5}{2}$ tablespoons of butter. She wants to make as many cookies as possible. To make one batch of cookies, Nicole needs $\frac{3}{8}$ tablespoon of butter. How many batches of cookies can she make?

In order to solve this problem, we need to take $\frac{5}{2}$ tablespoons and divide it by $\frac{3}{8}$ tablespoon.

Now let's learn how to divide so that we may determine the number of batches of cookies Nicole can make.

If you can multiply fractions, you should have no problem dividing fractions. We will use reciprocals to help us divide fractions. Multiply the first fraction by the reciprocal of the second fraction.

Division of Fractions

To divide fractions, multiply the first fraction by the reciprocal of the second fraction. (b, c, and d are not equal to zero.)

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{a \cdot d}{b \cdot c}$$

In our example Nicole would take:

$$\frac{5}{2} \div \frac{3}{8} = \frac{5}{2} \cdot \frac{8}{3} =$$

"long bar" $\quad \dfrac{5}{2} \cdot \dfrac{8}{3} = \dfrac{5 \cdot \overset{1}{\cancel{2}} \cdot 2 \cdot 2}{\underset{1}{\cancel{2}} \cdot 3} = \dfrac{20}{3}$

"cross canceling" $\quad \dfrac{5}{\underset{1}{\cancel{2}}} \cdot \dfrac{\overset{4}{\cancel{8}}}{3} = \dfrac{20}{3}$

So, $\frac{20}{3}$ is the same as $6\frac{2}{3}$. Nicole could make $6\frac{2}{3}$ batches of cookies.

Example 1

Find the quotient. Reduce to lowest terms. $\quad \frac{2}{3} \div \frac{5}{6}$

$$\frac{2}{3} \div \frac{5}{6} = \frac{2}{3} \cdot \frac{6}{5} =$$

"long bar" $\quad \dfrac{2}{3} \cdot \dfrac{6}{5} = \dfrac{2 \cdot 2 \cdot \overset{1}{\cancel{3}}}{\underset{1}{\cancel{3}} \cdot 5} = \dfrac{4}{5}$

"cross canceling" $\quad \dfrac{2}{\underset{1}{\cancel{3}}} \cdot \dfrac{\overset{2}{\cancel{6}}}{5} = \dfrac{4}{5}$

Example 2

Find the quotient. Reduce to lowest terms. $\quad 12 \div 4\frac{7}{8}$

$$12 \div 4\frac{7}{8} = \frac{12}{1} \div \frac{39}{8} = \frac{12}{1} \cdot \frac{8}{39} =$$

"long bar" $\quad \dfrac{12}{1} \cdot \dfrac{8}{39} = \dfrac{2 \cdot 2 \cdot \overset{1}{\cancel{3}} \cdot 2 \cdot 2 \cdot 2}{1 \cdot \underset{1}{\cancel{3}} \cdot 13} = \dfrac{32}{13}$

"cross canceling" $\quad \dfrac{\overset{4}{\cancel{12}}}{1} \cdot \dfrac{8}{\underset{13}{\cancel{39}}} = \dfrac{32}{13}$

Example 3

Find the quotient. Reduce to lowest terms. $\frac{x}{y^2z} \div \frac{x^2}{yz}$

When working with variables, it is best to use the "long bar" method.

$$\frac{x}{y^2z} \div \frac{x^2}{yz} = \frac{x}{y^2z} \cdot \frac{yz}{x^2} = \frac{\overset{1}{\cancel{x}} \cdot \overset{1}{\cancel{y}} \cdot \overset{1}{\cancel{z}}}{\underset{1}{\cancel{y}} \cdot y \cdot \underset{1}{\cancel{z}} \cdot \underset{1}{\cancel{x}} \cdot x} = \frac{1}{xy}$$

Example 4

Find the quotient. Reduce to lowest terms. $\dfrac{\frac{2}{3}}{\frac{3}{4}}$

$$\frac{\frac{2}{3}}{\frac{3}{4}} = \frac{2}{3} \div \frac{3}{4} = \frac{2}{3} \cdot \frac{4}{3} =$$

"long bar" $\quad \frac{2}{3} \cdot \frac{4}{3} = \frac{2 \cdot 2 \cdot 2}{3 \cdot 3} = \frac{8}{9}$

"cross canceling" $\quad \frac{2}{3} \cdot \frac{4}{3} = \frac{8}{9}$

Example 5

Find the quotient of $\frac{24}{25}$ and $\frac{12}{15}$. Reduce to lowest terms.

$$\frac{24}{25} \div \frac{12}{15} = \frac{24}{25} \cdot \frac{15}{12} =$$

"long bar" $\quad \frac{24}{25} \cdot \frac{15}{12} = \frac{\overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{2}} \cdot 2 \cdot \overset{1}{\cancel{3}} \cdot 3 \cdot \overset{1}{\cancel{5}}}{\underset{1}{\cancel{5}} \cdot 5 \cdot \underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{3}}} = \frac{6}{5}$

"cross canceling" $\quad \frac{\overset{2}{\cancel{24}}}{\underset{5}{\cancel{25}}} \cdot \frac{\overset{3}{\cancel{15}}}{\underset{1}{\cancel{12}}} = \frac{6}{5}$

Example 6

Divide $\frac{18}{35}$ into $\frac{9}{14}$. Reduce to lowest terms.

The word "into" reverses the order.

$$\frac{9}{14} \div \frac{18}{35} = \frac{9}{14} \cdot \frac{35}{18} = \left\langle \begin{array}{l} \text{"long bar"} \quad \frac{9}{14} \cdot \frac{35}{18} = \frac{\overset{1}{\cancel{3}} \cdot \overset{1}{\cancel{3}} \cdot 5 \cdot \overset{1}{\cancel{7}}}{2 \cdot \underset{1}{\cancel{7}} \cdot 2 \cdot \underset{1}{\cancel{3}} \cdot \underset{1}{\cancel{3}}} = \frac{5}{4} \\ \\ \text{"cross canceling"} \quad \frac{\overset{1}{\cancel{9}}}{\underset{2}{\cancel{14}}} \cdot \frac{\overset{5}{\cancel{35}}}{\underset{2}{\cancel{18}}} = \frac{5}{4} \end{array} \right.$$

Example 7

***Application*. If it takes $\frac{1}{2}$ cup of sugar to make a batch of cookies, how many batches of cookies can be made from $4\frac{3}{4}$ cups of sugar?**

To solve this problem, we must divide the total amount of sugar by the amount needed to make a batch of cookies.

$$4\frac{3}{4} \div \frac{1}{2} = \frac{19}{4} \div \frac{1}{2} = \frac{19}{4} \cdot \frac{2}{1} = \left\langle \begin{array}{l} \text{"long bar"} \quad \frac{19}{4} \cdot \frac{2}{1} = \frac{19 \cdot \overset{1}{\cancel{2}}}{\underset{1}{\cancel{2}} \cdot 2} = \frac{19}{2} \\ \\ \text{"cross canceling"} \quad \frac{19}{\underset{2}{\cancel{4}}} \cdot \frac{\overset{1}{\cancel{2}}}{1} = \frac{19}{2} \end{array} \right.$$

$\frac{19}{2}$ *or* $9\frac{1}{2}$ **batches of cookies could be made from $4\frac{3}{4}$ cups of sugar.**

3.3 EXERCISES

In 1-5, find the reciprocals of the fractions.

1. $\frac{5}{8}$

2. $7\frac{2}{3}$

3. -12

4. $\frac{x}{y}$

5. $-\frac{3}{4}$

In 6-18, divide the following fractions. Reduce all answers to lowest terms.

6. $\frac{17}{20} \div 13\frac{3}{5}$

7. $\frac{11}{8} \div \frac{11}{8}$

8. $-\frac{2}{3} \div \frac{5}{7}$

9. $\frac{4}{5} \div \frac{10}{2}$

10. $\dfrac{4\frac{1}{3}}{2\frac{8}{9}}$

11. $1\frac{1}{3} \div 8$

12. $\dfrac{\frac{14}{3}}{\frac{16}{9}}$

13. $-4 \div -\frac{2}{3}$

14. $\dfrac{7\frac{7}{8}}{9}$

15. $\frac{1}{4} \div \frac{1}{8}$

16. $\frac{ab^2}{cd} \div \frac{c^2d^2}{ab^2}$

17. $\frac{4x}{5y} \div \frac{3y}{25} \div \frac{7}{12x}$

18. $\frac{1890x^2yz^3}{924yz} \div \frac{30xyz}{231x^2yz}$

In 19-22, perform the indicated operation. Reduce all answers to lowest terms.

19. Find the quotient of $\frac{5}{6}$ and $\frac{15}{30}$.

20. What is the quotient of $\frac{3}{4}$ and -3?

21. Divide $\frac{25}{36}$ by $\frac{5}{12}$.

22. Divide $28\frac{1}{2}$ by 19.

Applications.

23. It takes $\frac{4}{5}$ of a yard of fabric to make a pillow. How many pillows can be made from $4\frac{2}{5}$ yards of fabric?

24. How many $\frac{1}{8}$ cup measures are there in $2\frac{3}{4}$ cups?

25. Kolby earned $38\frac{1}{2}$ dollars for working 7 hours. How much did he earn per hour? Express your answer in fraction form.

26. How many bottles containing $\frac{4}{5}$ pint of liquid can be filled from a 22-pint container?

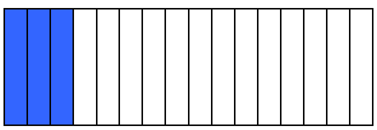
27. If a plane flies $65\frac{2}{3}$ miles in $7\frac{1}{2}$ minutes, how far can it travel in one minute?

3.4 Addition & Subtraction of Fractions

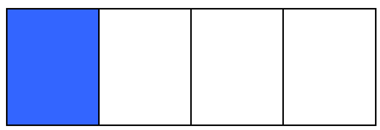
Now that we have learned how to multiply and divide fractions, we will focus our attention on addition and subtraction of fractions. To demonstrate how to add and subtract fractions we would like to use the following example:

In a mathematics class, students must earn a C to pass the class. If $\frac{3}{16}$ earned A's, $\frac{1}{4}$ earned B's, and $\frac{3}{8}$ earned C's, how many students passed the class?

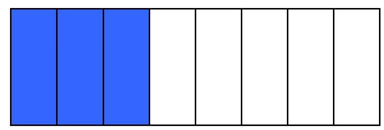
To find the number of students that passed the class, we must add the number of students that earned A's, B's, and C's. To do this, we will look at the following diagrams.
It is difficult to add the shaded regions because they are all different sizes. We will divide each original rectangle into sixteen smaller rectangles.



$\frac{3}{16}$

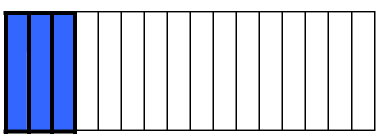


$\frac{1}{4}$

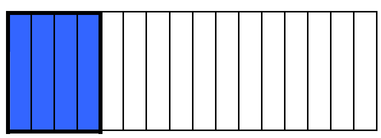


$\frac{3}{8}$

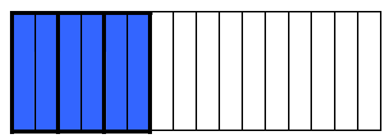
It is difficult to add the shaded regions because they are all different sizes. We will divide each original rectangle into sixteen smaller rectangles.



$\frac{3}{16}$



$\frac{1}{4}=\frac{4}{16}$



$\frac{3}{8}=\frac{6}{16}$

Now that the rectangles are all the same size, we can see there are a total of thirteen shaded rectangles. Therefore,

$$\frac{3}{16}+\frac{1}{4}+\frac{3}{8}=$$

$$\frac{3}{16}+\frac{4}{16}+\frac{6}{16}=\boxed{\frac{13}{16}}$$

13 out of 16 students passed the class.

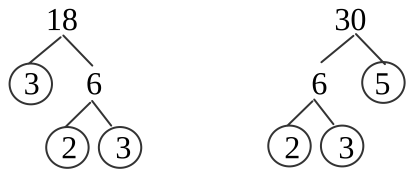
This process is called "finding the least common denominator".

The **least common denominator**, or **LCD**, of two or more fractions, is the smallest number that is a multiple of each denominator.

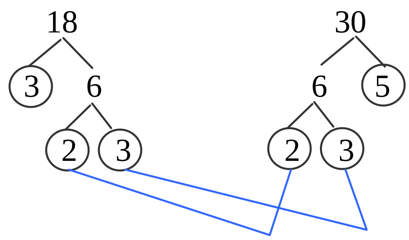
To find the LCD, we will demonstrate two methods. We used both of these methods to find the least common multiple (LCM) in Chapter 2. Method 1 used a factor tree and method 2 used the process known as repeated division.

Example 1 **Rewrite the fractions $\frac{11}{18}$ and $\frac{13}{30}$ with a LCD.**

Method 1 – Make a factor tree for each denominator.



Find the prime factors they have in common. (If you have three or more numbers, your common factors need to appear in at least two of them.)



In our problem, the numbers have a 2 and a 3 in common.

We will multiply the common factors, 2 and 3, along with any numbers that are not in common, in this case 3 and 5. So our LCD is:

$$\text{LCD} = 2 \cdot 3 \cdot 3 \cdot 5$$
$$\text{LCD} = 90$$

Method 2 – Repeated division.

To do repeated division, we write our denominators in the manner below.

$$\begin{array}{l|ll} & 18 & 30 \\ \hline \end{array}$$

Now we want to find the smallest prime number that will divide evenly into one or more of the denominators. Then divide and carry down the quotient. We repeat this process until you are left with ones across the bottom. If you choose a prime number that only divides evenly into one of the numbers, bring down the other number.

$$\begin{array}{r|rr} 2 & 18 & 30 \\ \hline 3 & 9 & 15 \\ \hline 3 & 3 & 5 \\ \hline 5 & 1 & 5 \\ \hline & 1 & 1 \end{array}$$

To find the LCD, multiply the numbers on the left of the repeated division.

$$\text{LCD} = 2 \cdot 3 \cdot 3 \cdot 5$$
$$\text{LCD} = 90$$

We found the LCD to be 90 regardless the method we used. Rewrite each fraction with that LCD. To do this we must find a form of 1 we can multiply the fraction by to get the denominator of 90. The first fraction has the denominator of 18. What can we multiply 18 by to get 90? To find this number, use division. $90 \div 18 = 5$. Our 1 will take the form of $\frac{5}{5}$. Multiply $\frac{11}{18}$ by $\frac{5}{5}$. Follow the same steps for the second fraction. $90 \div 30 = 3$. Our 1 will take the form of $\frac{3}{3}$. Multiply $\frac{13}{30}$ by $\frac{3}{3}$.

$$\frac{11}{18} \cdot \frac{5}{5} \quad \text{and} \quad \frac{13}{30} \cdot \frac{3}{3}$$

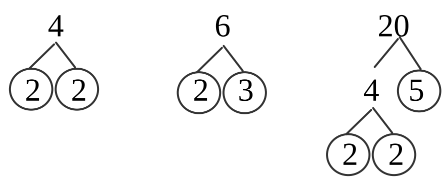
$$\frac{55}{90} \quad \text{and} \quad \frac{39}{90}$$

Now our fractions have a LCD of 90.

Example 2

Rewrite the fractions $\frac{3}{4}, \frac{5}{6}$, and $\frac{1}{20}$ with a LCD.

Method 1



$$\text{LCM} = 2 \cdot 2 \cdot 3 \cdot 5$$
$$\text{LCM} = 60$$

Method 2

$$\begin{array}{r|rrr} 2 & 4 & 6 & 20 \\ \hline 2 & 2 & 3 & 10 \\ \hline 3 & 1 & 3 & 5 \\ \hline 5 & 1 & 1 & 5 \\ \hline & 1 & 1 & 1 \end{array}$$

$$\text{LCM} = 2 \cdot 2 \cdot 3 \cdot 5$$
$$\text{LCM} = 60$$

$$\frac{3}{4} \cdot \frac{\overset{60 \div 4 = 15}{\downarrow} 15}{15} = \frac{45}{60} \qquad \frac{5}{6} \cdot \frac{\overset{60 \div 6 = 10}{\downarrow} 10}{10} = \frac{50}{60} \qquad \frac{1}{20} \cdot \frac{\overset{60 \div 20 = 3}{\downarrow} 3}{3} = \frac{3}{60}$$

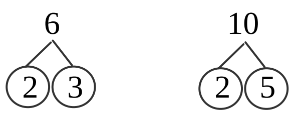
$$\frac{45}{60}, \frac{50}{60}, \text{ and } \frac{3}{60}$$

Now our fractions have a LCD of 60.

Example 3

Rewrite the fractions $\frac{1}{6}$ and $-\frac{7}{10}$ with a LCD.

Method 1



$LCD = 2 \cdot 3 \cdot 5$

$LCD = 30$

Method 2

$$\begin{array}{r|rr} 2 & 6 & 10 \\ \hline 3 & 3 & 5 \\ \hline 5 & 1 & 5 \\ \hline & 1 & 1 \end{array}$$

$LCD = 2 \cdot 3 \cdot 5$

$LCD = 30$

$$\frac{1}{6} \cdot \frac{\overset{30 \div 6 = 5}{\downarrow}\; 5}{5} = \frac{5}{30} \qquad -\frac{7}{10} \cdot \frac{\overset{30 \div 10 = 3}{\downarrow}\; 3}{3} = -\frac{21}{30}$$

$$\frac{5}{30} \text{ and } -\frac{21}{30}$$

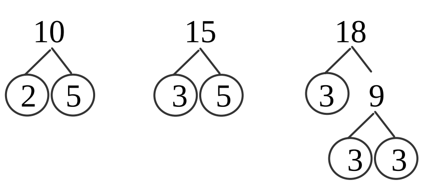
Example 4

Rewrite the fractions $2\frac{9}{10}, \frac{4}{15},$ and $1\frac{5}{18}$ with a LCD.

$$\frac{29}{10}, \frac{4}{15}, \textit{and } \frac{23}{18}$$

First, make the mixed numbers into improper fractions.

Method 1



$LCD = 2 \cdot 3 \cdot 3 \cdot 5$

$LCD = 90$

Method 2

$$\begin{array}{r|rrr} 2 & 10 & 15 & 18 \\ \hline 3 & 5 & 15 & 9 \\ \hline 3 & 5 & 5 & 3 \\ \hline 5 & 5 & 5 & 1 \\ \hline & 1 & 1 & 1 \end{array}$$

$LCD = 2 \cdot 3 \cdot 3 \cdot 5$

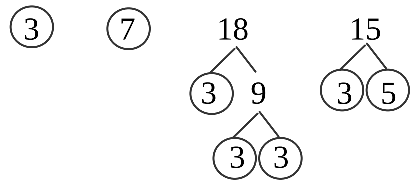
$LCD = 90$

$$\frac{29}{10} \cdot \frac{\overset{90 \div 10 = 9}{\downarrow}\; 9}{9} = \frac{261}{90} \qquad \frac{4}{15} \cdot \frac{\overset{90 \div 15 = 6}{\downarrow}\; 6}{6} = \frac{24}{90} \qquad \frac{23}{18} \cdot \frac{\overset{90 \div 18 = 5}{\downarrow}\; 5}{5} = \frac{115}{90}$$

$$\frac{261}{90}, \frac{24}{90}, \text{ and } \frac{115}{90}$$

Example 5 **Rewrite the fractions $-\frac{1}{3}, \frac{2}{7}, -\frac{5}{12}$, and $\frac{3}{14}$ with a LCD.**

Method 1



$$LCD = 2 \cdot 2 \cdot 3 \cdot 7$$
$$LCD = 84$$

Method 2

$$\begin{array}{r|rrrr} 2 & 3 & 7 & 12 & 14 \\ \hline 2 & 3 & 7 & 6 & 7 \\ \hline 3 & 3 & 7 & 3 & 7 \\ \hline 7 & 1 & 7 & 1 & 7 \\ \hline & 1 & 1 & 1 & 1 \end{array}$$

$$LCD = 2 \cdot 2 \cdot 3 \cdot 7$$
$$LCD = 84$$

$$-\frac{1}{3} \cdot \frac{\overset{84 \div 3 = 28}{\downarrow}\,28}{28} = -\frac{28}{84} \qquad \frac{2}{7} \cdot \frac{\overset{84 \div 7 = 12}{\downarrow}\,12}{12} = \frac{24}{84} \qquad -\frac{5}{12} \cdot \frac{\overset{84 \div 12 = 7}{\downarrow}\,7}{7} = -\frac{35}{84} \qquad \frac{3}{14} \cdot \frac{\overset{84 \div 14 = 6}{\downarrow}\,6}{6} = \frac{18}{84}$$

$$-\frac{28}{84}, \quad \frac{24}{84}, \quad -\frac{35}{84}, \quad \text{and} \quad \frac{18}{84}$$

**If you are having difficulty finding the LCD, you can find a common denominator, not necessarily the LCD, by multiplying the denominators of each fraction together. If you do this, you will most likely be required to do additional reducing of your final answer.

Now that we know how to rewrite our fractions with the least common denominator, let's look at the steps for addition and subtraction of fractions.

ADDITION AND SUBTRACTION OF FRACTIONS

If you have a common denominator,

1. Add or subtract the **numerators**, write that sum or difference over the common denominator.
2. Reduce if possible.

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c} \qquad\qquad \frac{a}{c} - \frac{b}{c} = \frac{a-b}{c}$$

If you do **not** have a common denominator,

1. Write each fraction with a common denominator.
2. Add or subtract the **numerators**, write that sum or difference over the common denominator.
3. Reduce, if possible.

Example 6

Find the difference. Reduce to lowest terms. $\frac{8}{11}-\frac{5}{11}$

$$\frac{8}{11}-\frac{5}{11} = \frac{8-5}{11}$$

There is a common denominator, so just find the difference.

$$= \frac{3}{11}$$

Example 7

Find the sum. Reduce to lowest terms. $1\frac{7}{12}+\frac{5}{18}$

$$1\frac{7}{12}+\frac{5}{18} = \frac{19}{12}+\frac{5}{18}$$

Rewrite the mixed number as an improper fraction.

$$= \frac{19}{12}\cdot\frac{3}{3}+\frac{5}{18}\cdot\frac{2}{2}$$

Find a common denominator.

$$= \frac{57}{56}+\frac{10}{36}$$

$$= \frac{57+10}{36}$$

Add.

$$= \frac{67}{36} \quad or \quad 1\frac{31}{36}$$

Example 8

Find the sum. Reduce to lowest terms. $-\frac{5}{12}+\frac{-1}{4}$

$$-\frac{5}{12}+\frac{-1}{4} = -\frac{5}{12}+\frac{-1}{4}\cdot\frac{3}{3}$$

Find a common denominator.

$$= \frac{-5}{12}+\frac{-3}{12} = \frac{-5+-3}{12}$$

Add.

$$= \frac{57}{56}+\frac{10}{36}$$

$$= \frac{\overset{-2}{\cancel{-8}}}{\underset{3}{\cancel{12}}}$$

Reduce.

$$= \frac{-2}{3}$$

Example 9

Find the difference. Reduce to lowest terms. $2-\frac{3}{4}$

$$2-\frac{3}{4} = \frac{2}{1}-\frac{3}{4}$$ *Rewrite the whole number as an improper fraction.*

$$= \frac{2}{1}\cdot\frac{4}{4}-\frac{3}{4}$$ *Find a common denominator.*

$$= \frac{8}{4}-\frac{3}{4}$$

$$= \frac{8-3}{4}$$ *Subtract.*

$$= \frac{5}{4} \quad or \quad 1\frac{1}{4}$$

Example 10

Perform the indicated operations. Reduce to lowest terms.

$$\frac{1}{2}+\frac{2}{3}+\frac{3}{4}$$

$$\frac{1}{2}+\frac{2}{3}+\frac{3}{4} = \frac{1}{2}\cdot\frac{6}{6}+\frac{2}{3}\cdot\frac{4}{4}+\frac{3}{4}\cdot\frac{3}{3}$$ *Find a common denominator.*

$$= \frac{6}{12}+\frac{8}{12}+\frac{9}{12}$$

$$= \frac{6+8+9}{12}$$ *Add.*

$$= \frac{23}{12} \quad or \quad 1\frac{11}{12}$$

Example 11

Perform the indicated operations. Reduce to lowest terms.

$$\frac{7}{12}+\frac{11}{15}-\frac{7}{18}$$

$$\frac{7}{12}+\frac{11}{15}-\frac{7}{18} = \frac{7}{12}\cdot\frac{15}{15}+\frac{11}{15}\cdot\frac{12}{12}-\frac{7}{18}\cdot\frac{10}{10}$$

Find a common denominator.

$$= \frac{105}{180}+\frac{132}{180}-\frac{70}{180}$$

$$= \frac{105+132-70}{180}$$

Add and subtract in order from left to right..

$$= \frac{167}{180}$$

Example 12

***Application*. Calissa runs $\frac{7}{8}$ of a mile. Tyler runs $\frac{7}{12}$ of a mile. Daisy runs $\frac{1}{4}$ of a mile. What is the sum of the distance they ran?**

To solve this problem, we must add all three fractions together.

$$\frac{7}{8}+\frac{7}{12}+\frac{1}{4}$$

$$\frac{7}{8}+\frac{7}{12}+\frac{1}{4} = \frac{7}{8}\cdot\frac{3}{3}+\frac{7}{12}\cdot\frac{2}{2}+\frac{1}{4}\cdot\frac{6}{6}$$

Find a common denominator.

$$= \frac{21}{24}+\frac{14}{24}+\frac{6}{24}$$

$$= \frac{21+14+6}{24}$$

Add.

$$= \frac{41}{24} \quad or \quad 1\frac{17}{24}$$

The sum of the distance ran is $\frac{41}{24}$ miles or $1\frac{17}{24}$ miles.

3.4 EXERCISES

In 1-8, rewrite fractions with a LCD.

1. $\frac{2}{5}$ *and* $-\frac{7}{15}$

2. $\frac{9}{11}, \frac{5}{6},$ *and* $\frac{2}{3}$

3. $\frac{3}{5}$ *and* $\frac{4}{9}$

4. $\frac{-11}{15}$ *and* $\frac{-5}{12}$

5. $\frac{1}{15}, \frac{3}{20},$ *and* $\frac{5}{24}$

6. $\frac{11}{12}$ *and* $\frac{3}{14}$

7. $\frac{13}{30}, \frac{11}{18},$ *and* $\frac{1}{24}$

8. $\frac{1}{2}, \frac{1}{3}, \frac{1}{4},$ *and* $\frac{1}{5}$

In 9-20, perform the indicated operations. Reduce all answers to lowest terms.

9. $-\frac{7}{10}-\frac{17}{10}$

10. $\frac{2}{3}-\left(-\frac{1}{3}\right)$

11. $\frac{4}{5}+\left(-\frac{5}{6}\right)$

12. $\frac{2}{3}+\frac{1}{4}$

13. $\frac{7}{8}-\frac{5}{7}$

14. $\frac{-4}{15}+\frac{17}{45}$

15. $2\frac{1}{3}+1\frac{2}{9}$

16. $4\frac{1}{5}+\frac{4}{15}$

17. $\frac{5}{6}+\frac{7}{12}+\frac{1}{24}$

18. $10\frac{1}{2}-3\frac{5}{6}$

19. $\left(-\frac{13}{12}\right)-\frac{2}{3}$

20. $\frac{6}{5}-\frac{1}{15}-\frac{1}{3}$

In 21-24, perform the indicated operation. Reduce all answers to lowest terms.

21. What is the sum of $\frac{2}{3}, \frac{4}{5}, \textit{and } \frac{11}{15}$?

22. Find the difference between $\frac{2}{9} \textit{ and } \frac{5}{6}$.

23. Find the sum of $\frac{9}{4} \textit{ and } -\frac{3}{2}$.

24. What is the difference between $\frac{9}{10} \textit{ and } \frac{2}{15}$?

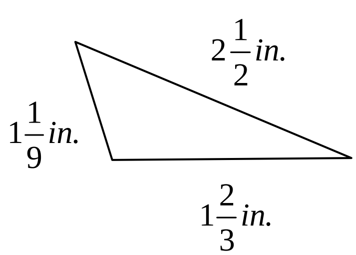
Applications.

25. A recipe calls for one and one-half cups of sugar for a cake and two-thirds cup of sugar for the frosting. How many cups of sugar does it take to make the entire recipe?

26. Cindee is trying to get into shape. She runs one-half of a mile on Monday, two-thirds of a mile on Tuesday, three-fourths of a mile on Wednesday, five-sixth of a mile on Thursday and five-fourths miles on Friday. How far did she run this week?

27. The Ison family spends one-third of their income on rent and one-sixth of their income on food. What fraction of their income is left?

28. The distance around a triangle is called the perimeter. To find the perimeter of a triangle, add the lengths of the sides. What is the perimeter of the following triangle?



29. Alesha bought fifteen cups of flour. She made cookies and used two and one-third cups. Then she made a dinner dish using one and one-half cups. She then made a loaf of bread to go with the dinner. The bread required four and five-sixths cups of flour. How much flour does Alesha have left?

3.5 Order of Operations with Fractions & Complex Fractions

ORDER OF OPERATIONS

Throughout this chapter, we have worked with expressions involving fractions. Each section has focused on a particular mathematical operation. This gave us the opportunity to learn how to work with fractions. Now we will face the challenge of dealing with numerous mathematical operations within a single expression. To simplify fraction expressions that involve more than one of these operations, apply the order of operations. **Always remember to reduce your final answer to lowest terms.**

ORDER OF OPERATIONS

1. Simplify within **parentheses** ( ) and other grouping symbols, such as **brackets** [ ], **braces** { }, or the **fraction bar —**. *(When more than one pair of grouping symbols occur within a problem, work the innermost set of grouping symbols first.)*
2. Evaluate an expression involving **exponents** or **roots**.
3. **Multiply** or **divide** in order from left to right.
4. **Add** or **subtract** in order from left to right.

Example 1 **Simplify the following expression:** $\frac{3}{8}\cdot\frac{5}{9}+\frac{3}{4}$

$$\frac{3}{8}\cdot\frac{5}{9}+\frac{3}{4} = \frac{\cancel{3}^{1}}{8}\cdot\frac{5}{\cancel{9}_{3}}+\frac{3}{4} = \frac{5}{24}+\frac{3}{4}$$

Multiply first.

$$= \frac{5}{24}+\frac{3}{4}\cdot\frac{6}{6} = \frac{5}{24}+\frac{18}{24}$$

Find a common denominator, then add.

$$= \frac{23}{24}$$

Example 2 **Simplify the following expression:** $\frac{2}{3}+\left(-\frac{1}{4}\right)^2$

$$\frac{2}{3}+\left(-\frac{1}{4}\right)^2 = \frac{2}{3}+\frac{1}{16}$$

Simplify exponents., Note: a negative times a negative is a positive.

$$= \frac{2}{3}\cdot\frac{16}{16}+\frac{1}{16}\cdot\frac{3}{3} = \frac{32}{48}+\frac{3}{48}$$

Find a common denominator, then add.

$$= \frac{35}{48}$$

Example 3

Simplify the following expression: $2\frac{1}{2}+\frac{1}{3}(6+2)$

$$2\frac{1}{2}+\frac{1}{3}(6+2) = \frac{5}{2}+\frac{1}{3}(6+2)$$

Change the mixed number into an improper fraction. Follow the order of operations by simplifying within the parentheses.

$$= \frac{5}{2}+\frac{1}{3}(8)$$

$$= \frac{5}{2}+\frac{1}{3}\left(\frac{8}{1}\right) = \frac{5}{2}+\frac{8}{3}$$

Rewrite 8 as an improper fraction and multiply.

$$= \frac{5}{2}\cdot\frac{3}{3}+\frac{8}{3}\cdot\frac{2}{2} = \frac{15}{6}+\frac{16}{6}$$

Find a common denominator then, add.

$$= \frac{31}{6} \quad or \quad 5\frac{1}{6}$$

Example 4

Simplify the following expression: $2\frac{1}{4}-1\frac{1}{2}\cdot\frac{3}{4}$

$$2\frac{1}{4}-1\frac{1}{2}\cdot\frac{3}{4} = \frac{9}{4}-\frac{3}{2}\cdot\frac{3}{4}$$

Change the mixed numbers into improper fractions. Follow the order of operations and multiply

$$= \frac{9}{4}-\frac{9}{8} = \frac{9}{4}\cdot\frac{2}{2}-\frac{9}{8} = \frac{18}{8}-\frac{9}{8}$$

Find a common denominator then, add.

$$= \frac{9}{8} \quad or \quad 1\frac{1}{8}$$

Example 5

Simplify the following expression: $\sqrt{\frac{16}{25}}\left[\left(\frac{2}{3}\right)^2 - \frac{1}{2}\cdot\frac{3}{4}\right]$

$$\sqrt{\frac{16}{25}}\left[\left(\frac{2}{3}\right)^2 - \frac{1}{2}\cdot\frac{3}{4}\right] = \sqrt{\frac{16}{25}}\left[\frac{4}{9} - \frac{1}{2}\cdot\frac{3}{4}\right]$$

Follow the order of operations by simplifying within the parentheses.

$$= \sqrt{\frac{16}{25}}\left[\frac{4}{9} - \frac{3}{8}\right]$$

$$= \sqrt{\frac{16}{25}}\left[\frac{4}{9}\cdot\frac{8}{8} - \frac{3}{8}\cdot\frac{9}{9}\right] = \sqrt{\frac{16}{25}}\left[\frac{32}{72} - \frac{27}{72}\right]$$

Find a common denominator then, subtract.

$$= \sqrt{\frac{16}{25}}\left[\frac{5}{72}\right]$$

Simplify the square root.

$$= \frac{\overset{1}{\cancel{4}}}{\underset{1}{\cancel{5}}}\left[\frac{\overset{1}{\cancel{5}}}{\underset{18}{\cancel{72}}}\right]$$

Multiply and reduce.

$$= \frac{1}{18}$$

Example 6

Simplify the following expression: $\left(\frac{1}{2} + \frac{2}{3}\right)\left(\frac{1}{7} + \frac{5}{7}\right)$

$$\left(\frac{1}{2} + \frac{2}{3}\right)\left(\frac{1}{7} + \frac{5}{7}\right) = \left(\frac{1}{2}\cdot\frac{3}{3} + \frac{2}{3}\cdot\frac{2}{2}\right)\left(\frac{1}{7} + \frac{5}{7}\right)$$

Follow the order of operations by simplifying within the parentheses.

$$= \left(\frac{3}{6} + \frac{4}{6}\right)\left(\frac{1}{7} + \frac{5}{7}\right)$$

Add.

$$= \left(\frac{\overset{1}{\cancel{7}}}{\underset{1}{\cancel{6}}}\right)\left(\frac{\overset{1}{\cancel{6}}}{\underset{1}{\cancel{7}}}\right)$$

Multiply and reduce.

$$= 1$$

Complex Fractions

Complex fractions sound difficult, but they are not as complex as they sound. A **complex fraction** is a fraction within a fraction. The following are examples of complex fractions:

$$\frac{\frac{2}{5}}{\frac{7}{10}}, \quad \frac{3}{\frac{4}{5}}, \quad \frac{\frac{1}{2}-\frac{1}{3}}{\frac{3}{7}}$$

To simplify a complex fraction start by rewriting the expression as a division problem. If the numerator or denominator contains more than one term, enclose them in a set of parentheses. Follow your order of operations to simplify the expression.

Example 7 **Simplify the following expression:** $\frac{\frac{3}{10}}{\frac{1}{2}}$

$$\frac{\frac{3}{10}}{\frac{1}{2}} = \frac{3}{10} \div \frac{1}{2}$$ *Rewrite as a division problem.*

$$= \frac{3}{10} \cdot \frac{2}{1} = \frac{3}{\cancel{10}_{5}} \cdot \frac{\cancel{2}^{1}}{1}$$ *Invert and multiply. Then reduce.*

$$= \frac{3}{5}$$

Example 8 **Simplify the following expression:** $\frac{2\frac{1}{2}}{9}$

$$\frac{2\frac{1}{2}}{9} = 2\frac{1}{2} \div 9$$ *Rewrite as a division problem.*

$$= \frac{5}{2} \div \frac{9}{1}$$ *Change the mixed number and the whole number to improper fractions.*

$$= \frac{5}{2} \cdot \frac{1}{9}$$ *Invert and multiply.*

$$= \frac{5}{18}$$

Example 9 **Simplify the following expression:** $\dfrac{\frac{2}{5}+\frac{1}{6}}{\frac{3}{4}-\frac{7}{8}}$

$$\frac{\frac{2}{5}+\frac{1}{6}}{\frac{3}{4}-\frac{7}{8}} = \left(\frac{2}{5}+\frac{1}{6}\right)\div\left(\frac{3}{4}-\frac{7}{8}\right)$$

Rewrite as a division problem. Use parentheses for both the numerator and denominator.

$$= \left(\frac{2}{5}\cdot\frac{6}{6}+\frac{1}{6}\cdot\frac{5}{5}\right)\div\left(\frac{3}{4}\cdot\frac{2}{2}-\frac{7}{8}\right)$$

Simplify within each group of parentheses. Start by finding a LCD.

$$= \left(\frac{12}{30}+\frac{5}{30}\right)\div\left(\frac{6}{8}-\frac{7}{8}\right)$$

Add and Subtract.

$$= \left(\frac{17}{30}\right)\div\left(-\frac{1}{8}\right)$$

$$= \frac{17}{30}\cdot -\frac{8}{1} = \frac{17}{\underset{15}{\cancel{30}}}\cdot -\frac{\overset{4}{\cancel{8}}}{1}$$

Invert and multiply. Then reduce.

$$= -\frac{68}{15} \quad \text{or} \quad -4\frac{8}{15}$$

Example 10 **Simplify the following expression:** $\dfrac{5-\frac{3}{4}}{2-\frac{1}{3}}$

$$\frac{5-\frac{3}{4}}{2-\frac{1}{3}} = \left(5-\frac{3}{4}\right)\div\left(2-\frac{1}{3}\right) = \left(\frac{5}{1}-\frac{3}{4}\right)\div\left(\frac{2}{1}-\frac{1}{3}\right)$$

Rewrite as a division problem. Change the whole numbers, 5 and 2, into improper fractions. Use parentheses for both the numerator and denominator.

$$= \left(\frac{5}{1}\cdot\frac{4}{4}-\frac{3}{4}\right)\div\left(\frac{2}{1}\cdot\frac{3}{3}-\frac{1}{3}\right)$$

Simplify within each group of parentheses. Start by finding a LCD.

$$= \left(\frac{20}{4}-\frac{3}{4}\right)\div\left(\frac{6}{3}-\frac{1}{3}\right) = \left(\frac{17}{4}\right)\div\left(\frac{5}{3}\right)$$

Add and Subtract.

$$= \frac{17}{4}\cdot\frac{3}{5} = \frac{51}{20} \quad \text{or} \quad 2\frac{11}{20}$$

Invert and multiply.

Example 9

Simplify the following expression: $$\frac{\left(1\frac{1}{2}\right)^2}{\frac{2}{3}+\frac{2}{9}}$$

$$\frac{\left(1\frac{1}{2}\right)^2}{\frac{2}{3}+\frac{2}{9}} = \left(1\frac{1}{2}\right)^2 \div \left(\frac{2}{3}+\frac{2}{9}\right)$$

Rewrite as a division problem. Use parentheses for the denominator. Change the mixed number into an improper fraction.

$$= \left(\frac{3}{2}\right)^2 \div \left(\frac{2}{3}\cdot\frac{3}{3}+\frac{2}{9}\right)$$

Simplify within the parentheses. Start by finding a LCD.

$$= \left(\frac{3}{2}\right)^2 \div \left(\frac{6}{9}+\frac{2}{9}\right)$$

Add.

$$= \left(\frac{3}{2}\right)^2 \div \left(\frac{8}{9}\right)$$

Simplify the exponent.

$$= \left(\frac{9}{4}\right) \div \left(\frac{8}{9}\right)$$

$$= \frac{9}{4}\cdot\frac{9}{8}$$

Invert and multiply.

$$= \frac{81}{32} \quad or \quad 2\frac{17}{32}$$

3.5 EXERCISES

In 1-10, simplify the following expressions. Reduce all answers to lowest terms.

1. $\sqrt{\frac{1}{9}}+\left(\frac{2}{3}\right)^2$

2. $\frac{2}{3}+4\left(\frac{1}{2}+\frac{3}{2}\right)$

3. $\left(\frac{2}{3}\right)^2\left(\frac{3}{4}\right)-\frac{2}{5}$

4. $\left(\frac{3}{4}\right)\left(\frac{1}{2}\right)^2+\frac{1}{8}$

5. $\left(\frac{2}{3}+\frac{2}{9}\right)\left(\frac{1}{2}-\frac{2}{5}\right)$

6. $\frac{2}{3}\left(\frac{3}{4}+1\right)+\left(\frac{1}{3}\right)^2$

7. $\frac{5}{9}\div\left(-\frac{35}{39}\right)\left(1\frac{1}{13}\right)$

8. $\frac{4}{7}\div\left(-\frac{2}{7}\right)\left(\frac{-3}{8}\right)$

9. $\sqrt{\frac{9}{16}}\left[\frac{1}{2}-\left(\frac{3}{4}\right)^2\right]$

10. $1\frac{1}{6}\div\frac{1}{2}+1\frac{1}{5}-\frac{2}{3}$

In 11-20, simplify. Reduce all answers to lowest terms.

11. $$\frac{\frac{3}{4}}{\frac{7}{8}}$$

12. $$\frac{\frac{1}{3}}{\frac{5}{6}}$$

13. $$\frac{\frac{1}{8}+\frac{3}{4}}{\frac{1}{2}-\frac{1}{3}}$$

14. $$\frac{\frac{2}{3}+\frac{3}{2}}{\frac{1}{3}-\frac{5}{6}}$$

15. $$\frac{2\frac{1}{3}}{9}$$

16. $$\frac{\frac{3}{7}+5}{6-\frac{8}{14}}$$

17. $$\frac{\left(\frac{5}{2}\right)^2}{2-\frac{1}{2}}$$

18. $$\frac{7}{3\frac{5}{6}}$$

19. $$\frac{2\frac{1}{3}+\frac{2}{9}}{\left(\frac{2}{3}\right)^2}$$

20. $$\frac{7-\frac{1}{2}}{3-\frac{1}{8}}$$

3.6 REVIEW EXERCISES

In 1-6, identify the following as a proper fraction, improper fraction, or mixed number.

1. $3\frac{1}{7}$
2. $-\frac{11}{3}$
3. 4
4. $\frac{13}{13}$
5. $\frac{2}{3}$
6. $-12\frac{3}{8}$

In7-10, change the following improper fractions into mixed numbers using division.

7. $\frac{7}{4}$
8. $\frac{100}{3}$
9. $-\frac{19}{13}$
10. $\frac{28}{5}$

In 11-14, change the following mixed numbers into improper fractions.

11. $11\frac{1}{3}$
12. $-4\frac{2}{5}$
13. $2\frac{11}{13}$
14. $-7\frac{4}{9}$

In 15-20, reduce or simplify the following fractions to lowest terms.

15. $\frac{120}{200}$
16. $-\frac{24a^2b}{18ab^5}$
17. $-\frac{192}{264}$
18. $\frac{125x^4y^5z}{15x^0y^6z^2}$
19. $\frac{a^3bc^4d^0}{a^2b^3d}$
20. $\frac{160r^2st}{252rs^2t^4}$

In 21-38, perform the indicated operations. Remember to reduce your answer to lowest terms.

21. $2\frac{3}{4} \cdot \frac{8}{9}$

22. $\frac{a^2b}{c} \cdot \frac{c^3}{ab^2}$

23. Find the product of $\frac{18x^2}{y}$ and $\frac{y^2}{9x}$

24. $\frac{\frac{6}{7}}{\frac{3}{14}}$

25. $-\frac{5}{6} \div \frac{5}{6}$

26. Divide $\frac{1}{84}$ by $\frac{3}{49}$

27. $\frac{11}{20} \div \frac{3}{5x} \div \frac{40x^2}{12x}$

28. $\frac{1}{8} + \frac{5}{8}$

29. $\frac{7}{12} + \frac{3}{4}$

30. Find the sum of $\frac{1}{3}$ and $1\frac{5}{18}$

31. $3\frac{1}{4} - 1\frac{1}{2}$

32. $-2\frac{3}{4} - \frac{7}{12}$

33. Find the difference between $\frac{11}{27}$ and $\frac{1}{18}$

34. $\sqrt{\frac{9}{25}} \div \left(-\frac{3}{25}\right)\left(-\frac{3}{11}\right)$

35. $\frac{1}{3} + \frac{2}{9}\left(3\frac{1}{2} + 3\frac{1}{2}\right)$

36. $\left(\frac{2}{3}\right)^2 \left(\frac{3}{4}\right) - \frac{1}{5}$

37. $\frac{\frac{3}{4} + \frac{1}{3}}{\frac{2}{3} + \frac{1}{6}}$

38. $\frac{\frac{2}{7} + 5}{6 - \frac{5}{14}}$