2.1 The Factor Tree, The Greatest Common Factor (GCF) & The Least Common Multiple (LCM)

In chapter 1, we discussed the types of numbers and their basic mathematical operations. In this chapter, we will build upon those ideas as we learn additional definitions and operations dealing with those numbers. We will begin with the definitions of a few key mathematical terms.

Factors are the numbers multiplied together to obtain a given product. For example, 3 and 5 are factors of 15. 1 and 15 are also factors of 15.

A **prime number** is a whole number greater than one with exactly two factors, one and itself. The prime numbers to 100 include: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, …

A **composite number** is a whole number greater then one that is not prime.

THE FACTOR TREE

In mathematics, it is beneficial to know how to write a composite number as a product of prime numbers, also know as **prime factorization**. To do this, we will use what is known as a **factor tree.**

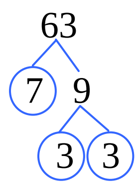
The rules for divisibility, found in figure 2.1 (below), will aid you in the process of prime factorization.

DIVISIBILITY TESTS

A number is divisible by **2** if it ends in 0, 2, 4, 6, or 8. (An even number.)
A number is divisible by **3** if the sum of its digits is divisible by 3.
A number is divisible by **5** if it ends in 0 or 5.

Figure 2.1

Use a factor tree to find the prime factorization of 63.



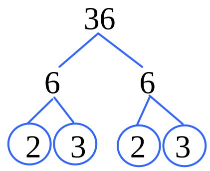
Start by finding two numbers whose product is 63. The number 7 is prime, circle it. The number 9 is not, so find two numbers whose product is 9.

The number 3 is prime, circle both threes. You are finished when the numbers at the bottom of each branch are prime numbers.

The prime factorization of $63 = 3 \cdot 3 \cdot 7$.

Example 1

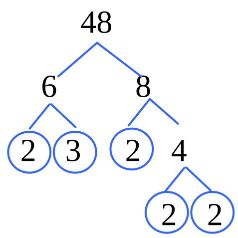
Find the prime factorization of 36.



The prime factorization of $36 = 2 \cdot 2 \cdot 3 \cdot 3$

Example 2

Find the prime factorization of 48.



The prime factorization of $48 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3$

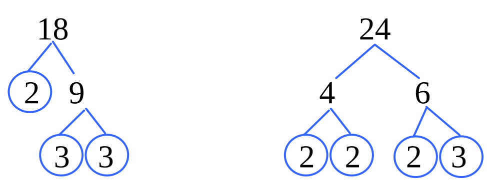
The Greatest Common Factor (GCF)

The **Greatest Common Factor (GCF)** is the largest factor that is the same in all the given numbers. (The largest number that can divide evenly into all the numbers.)

To find the greatest common factor, start by prime factoring each number. Then identify the common factors. If there is more than one common factor, the greatest common factor is the product of all the common factors. If there are no common factors, the greatest common factor is 1.

Example 3

Find the greatest common factor of 18 and 24.



Prime factor each number.

$2 \cdot 3 \cdot 3$ $\qquad$ $2 \cdot 2 \cdot 2 \cdot 3$

Write as a product of prime factors from least to greatest.

$\cancel{2} \cdot \cancel{3} \cdot 3$ $\qquad$ $\cancel{2} \cdot 2 \cdot 2 \cdot \cancel{3}$

Identify and write the common factors. (It may be helpful to cross out the common factors.) There are two factors in common.

$\text{GCF} = 2 \cdot 3$

$\text{GCF} = 6$

Multiply the common factors to get the greatest common factor.

Example 4

Find the greatest common factor of 10 and 21.

$10 \rightarrow (2)(5)$ $\quad$ $21 \rightarrow (3)(7)$

Prime factor each number.

$2 \cdot 5$ $\quad$ $3 \cdot 7$

Write as a product of prime factors from least to greatest.

$\text{GCF} = 1$

Identify and write the common factors. There are no common factors, so the greatest common factor is 1.

Example 5

Find the greatest common factor of 35 and 42.

$35 \rightarrow (5)(7)$ $\quad$ $42 \rightarrow (7)\ 6$, $6 \rightarrow (2)(3)$

Prime factor each number.

$5 \cdot 7$ $\quad$ $2 \cdot 3 \cdot 7$

Write as a product of prime factors from least to greatest.

$5 \cdot \not{7}$ $\quad$ $2 \cdot 3 \cdot \not{7}$

Identify and write the common factors.

$\text{GCF} = 7$

There is only one factor in common. The greatest common factor is that number.

Example 6

Find the greatest common factor of 9 and 18.

$9 \rightarrow (3)(3)$ $\quad$ $18 \rightarrow (2)\ 9$, $9 \rightarrow (3)(3)$

Prime factor each number.

$3 \cdot 3$ $\quad$ $2 \cdot 3 \cdot 3$

Write as a product of prime factors from least to greatest.

$\not{3} \cdot \not{3}$ $\quad$ $2 \cdot \not{3} \cdot \not{3}$

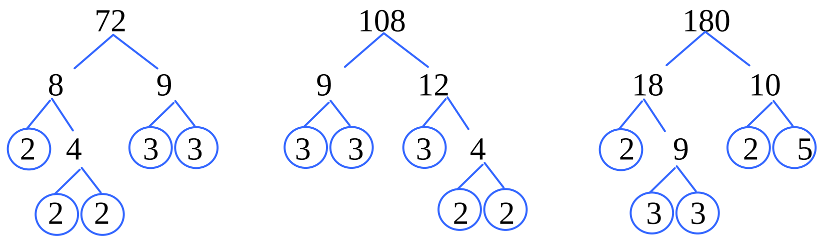
Identify and write the common factors.

$\text{GCF} = 3 \cdot 3$

$\text{GCF} = 9$

There are two factors in common. Multiply the common factors to get the greatest common factor.

Example 7 **Find the greatest common factor of 72, 108 and 180.**



Prime factor each number.

$2\cdot 2\cdot 2\cdot 3\cdot 3$ $\quad$ $2\cdot 2\cdot 3\cdot 3\cdot 3$ $\quad$ $2\cdot 2\cdot 3\cdot 3\cdot 5$

Write as a product of prime factors from least to greatest.

$\cancel{2}\cdot\cancel{2}\cdot 2\cdot\cancel{3}\cdot\cancel{3}$ $\quad$ $\cancel{2}\cdot\cancel{2}\cdot\cancel{3}\cdot\cancel{3}\cdot 3$ $\quad$ $\cancel{2}\cdot\cancel{2}\cdot\cancel{3}\cdot\cancel{3}\cdot 5$

Identify and write the common factors. There are four factors in common.

$$\text{GCF} = 2\cdot 2\cdot 3\cdot 3$$
$$\text{GCF} = 36$$

Multiply the common factors to get the greatest common factor.

THE LEAST COMMON MULTIPLE (LCM)

The **Least Common Multiple (LCM)** is the smallest number that is a multiple of each of the given number.

We will demonstrate two methods for finding the least common multiple. The first method uses factor tree, the second method uses what is known as repeated division.

Example 8 **Find the least common multiple of 4 and 10.**

Method 1 – Make a factor tree for each number.

Find the prime factors they have in common. (If you have three or more numbers, your common factors need to appear in at least two of the numbers.)

In our problem, the numbers have a 2 in common.

We will multiply the common factor, 2, along with any numbers that are not in common, in this case 2, and 5. Our least common multiple (LCM) is:

$$\text{LCM} = 2\cdot 2\cdot 5$$
$$\text{LCM} = 20$$

Method 2 – Repeated division.

To do repeated division, we write our numbers in the manner below.

$$\underline{\big|\,4 \quad 10}$$

Now we want to find the smallest prime number that will divide evenly into one or more of the numbers. Then divide and carry down the quotient. We repeat this process until you are left with 1's across the bottom. If you choose a prime number that only divides evenly into one of the numbers, bring down the other number.

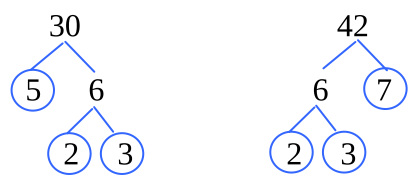
$$\begin{array}{r|rr} 2 & 4 & 10 \\ \hline 2 & 2 & 5 \\ \hline 5 & 1 & 5 \\ \hline & 1 & 1 \end{array}$$

To find the LCM, multiply the numbers to the left of the repeated division.

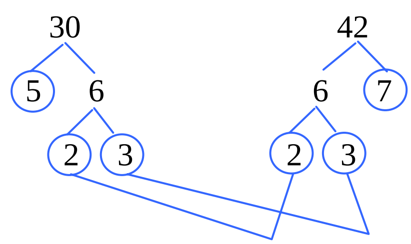
$$\text{LCM} = 2 \cdot 2 \cdot 5$$
$$\text{LCM} = 20$$

Example 9 **Find the least common multiple of 30 and 42.**

Method 1 – Make a factor tree for each number.



Find the prime factors they have in common. (If you have three or more numbers, your common factors need to appear in at least two of the numbers.)



In our problem, the numbers have a 2 and a 3 in common.

We will multiply the common factors, 2 and 3, along with any numbers that are not in common, in this case 5, and 7. Our least common multiple (LCM) is:

$$\text{LCM} = 2 \cdot 3 \cdot 5 \cdot 7$$
$$\text{LCM} = 210$$

Method 2 – Repeated division.

To do repeated division, we write our numbers in the manner below.

$$\begin{array}{r|rr} & 30 & 42 \\ \hline \end{array}$$

Now we want to find the smallest prime number that will divide evenly into one or more of the numbers. Then divide and carry down the quotient. We repeat this process until you are left with 1's across the bottom. If you choose a prime number that only divides evenly into one of the numbers, bring down the other number.

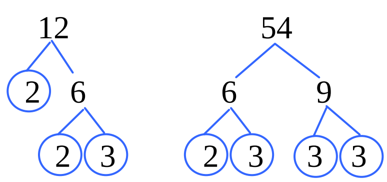
$$\begin{array}{r|rr} 2 & 30 & 42 \\ \hline 3 & 15 & 21 \\ \hline 5 & 5 & 7 \\ \hline 7 & 1 & 7 \\ \hline & 1 & 1 \end{array}$$

To find the LCM, multiply the numbers to the left of the repeated division.

$$\text{LCM} = 2 \cdot 3 \cdot 5 \cdot 7$$
$$\text{LCM} = 210$$

Example 10 **Find the least common multiple of 12 and 54.**

Method 1



$$\text{LCM} = 2 \cdot 3 \cdot 2 \cdot 3 \cdot 3$$
$$\text{LCM} = 108$$

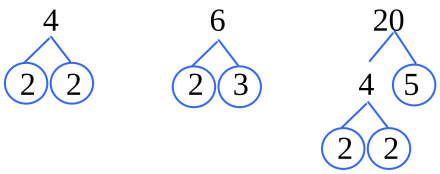
Method 2

$$\begin{array}{r|rr} 2 & 12 & 54 \\ \hline 2 & 6 & 27 \\ \hline 3 & 3 & 27 \\ \hline 3 & 1 & 9 \\ \hline 3 & 1 & 3 \\ \hline & 1 & 1 \end{array}$$

$$\text{LCM} = 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3$$
$$\text{LCM} = 108$$

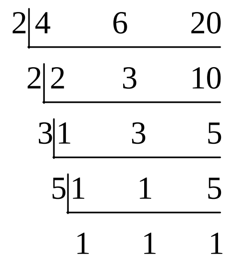
Example 11 **Find the least common multiple of 4, 6, and 20.**

Method 1



$$LCM = 2 \cdot 2 \cdot 3 \cdot 5$$
$$LCM = 60$$

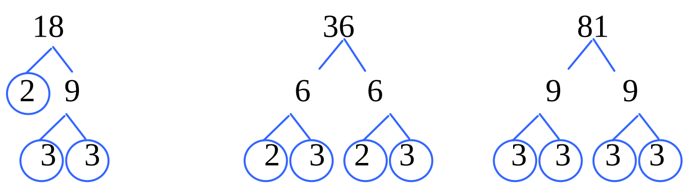
Method 2



$$LCM = 2 \cdot 2 \cdot 3 \cdot 5$$
$$LCM = 60$$

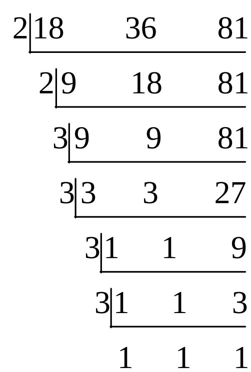
Example 12 **Find the least common multiple of 18, 36, and 81.**

Method 1



$$LCM = 2 \cdot 3 \cdot 3 \cdot 2 \cdot 3 \cdot 3$$
$$LCM = 324$$

Method 2



$$LCM = 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3 \cdot 3$$
$$LCM = 324$$

Note: To help you differentiate between the greatest common factor (GCF) and the least common multiple (LCM) you can think that both include the common factors but the **L**east common multiple also includes the **L**eft over factors. (Least common multiple and Leftovers both start with **L**.)

Example 13

Find the greatest common factor and least common multiple of 6 and 15.

Start by prime factoring each number.

Write as a product of prime factors from least to greatest. $2 \cdot 3$ $3 \cdot 5$

Greatest common factor:

$\text{GCF} = 3$

Identify and write the common factors. There is only one factor in common. The greatest common factor is that number.

Least common multiple:

3 is a common factor

Identify the common prime factors.

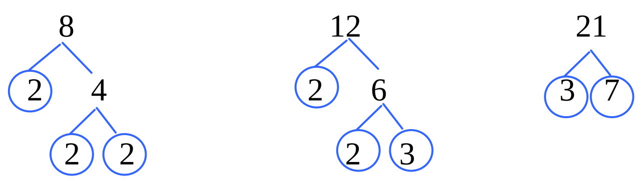
$$\text{LCM} = 3 \cdot 2 \cdot 5$$
$$\text{LCM} = 30$$

Multiply the common prime factors by any remaining prime factors.

Example 14

Find the greatest common factor and least common multiple of 8, 12, and 21.

Start by prime factoring each number.



Write as a product of prime factors from least to greatest. $2 \cdot 2 \cdot 2$ $2 \cdot 2 \cdot 3$ $3 \cdot 7$

Greatest common factor:

$\text{GCF} = 1$

Identify and write the common factors. There are no common factors in all three numbers, so the greatest common factor is 1.

Least common multiple:

$2 \cdot 2 \cdot 3$ are common factor

Identify the common prime factors in at least two of the numbers.

$$\text{LCM} = 2 \cdot 2 \cdot 3 \cdot 2 \cdot 7$$
$$\text{LCM} = 168$$

Multiply the common prime factors by any remaining prime factors.

2.1 EXERCISES

In 1-8, use a factor tree to write the prime factorization of each number.

1. 15
2. 12
3. 45
4. 80
5. 13
6. 240
7. 210
8. 1000

In 9-20, find the greatest common factor (GCF) of the following sets of numbers.

9. 8 and 12
10. 24 and 40
11. 12 and 35
12. 18 and 42
13. 9 and 10
14. 12 and 48
15. 120 and 216
16. 15, 20, and 30
17. 40, 50, and 60
18. 18, 30, and 42
19. 72, 108, and 180
20. 700, 420, and 1,120

In 21-32, find the least common multiple (LCM) of the following sets of numbers.

21. 3 and 4
22. 4 and 6
23. 25 and 80
24. 9 and 15
25. 12 and 20
26. 15 and 25
27. 4, 6, and 18
28. 3, 21, and 56
29. 70, 80, and 90
30. 10, 15, and 100
31. 7, 12, and 28
32. 11, 33, and 121

In 33-36, find the GCF and the LCM of the following sets of numbers.

33. 12 and 18
34. 45 and 60
35. 6, 8, and 24
36. 10, 25, and 30

2.2 Exponents and Roots

EXPONENTS

In mathematics, there are shorthand ways of writing things. **Exponents** are an example of a shorthand way to write repeated multiplication. An exponent can also be referred to as a power of the base. For example, $3 \times 3 \times 3 \times 3 \times 3$ can be written as 3^5. (3 multiplied by itself 5 times.) The number 3 is called the base and the number 5 is called the exponent. ($3 \times 3 \times 3 \times 3 \times 3$ is considered to be the *expanded form* of a base raised to an exponent.)

Exponents are not only used with numbers. They can be used with the letters of mathematics, called **variables**. For example, $x \cdot x \cdot x \cdot x$ can be written as x^4, where the variable, *x*, is the base and the number 4 is the exponent. An important concept to understand when working with numbers and variables occurs when a number is placed before a variable as in 5*x*. The operation performed is multiplication. 5*x* is the same as 5 times *x*.

How do exponents affect negative numbers? Look at the following examples of the exponents written in expanded form.

$$\begin{aligned}(-2)^4 &= (-2)(-2)(-2)(-2)\\ &= 16\end{aligned}$$

When the negative is in the parentheses, the negative is included in the repeated multiplication.

$$\begin{aligned}-2^4 &= -2 \cdot 2 \cdot 2 \cdot 2\\ &= -16\end{aligned}$$

When the negative is not in the parentheses, the negative is not included in the repeated multiplication. It is written once in the front.

Numbers and variables with no exponents showing are understood to have an exponent of 1. In the example, $3x^2$, the 3 has an exponent of 1 and the *x* has an exponent of 2, allowing us to write it in expanded form as $3 \cdot x \cdot x$.

The most common error that occurs when working with examples such as the one above is to forget the fact that if an exponent is not showing it is understood to be a 1. This error causes some to attach the exponent on the variable to both the number and the variable. In our example, $3x^2$, this error is seen when the 3 is written twice and the variable is written twice. The proper way to write $3x^2$ in expanded form is $3 \cdot x \cdot x$ not ~~$3 \cdot 3 \cdot x \cdot x$~~. Recall, numbers and variables with no exponent showing are understood to have an exponent of **1**.

Example 1 **Rewrite $4 \cdot 4 \cdot 4$ with an exponent.**

4^3

4 multiplied by itself 3 times.

Example 2 **Rewrite $(-5)(-5)(-5)(-5)(-5)(-5)$ with an exponent.**

$(-5)^6$

–5 multiplied by itself 6 times. The negative must be enclosed in the parentheses because it is included in the repeated multiplication.

Example 3

Rewrite $-3 \cdot 3 \cdot x \cdot x \cdot x \cdot x$ with an exponent.

$-3^2 x^4$

*The negative is **not** enclosed in the parentheses because it is **not** included in the repeated multiplication.*

Example 4

Rewrite 11 with an exponent.

11^1

A number and/or a variable written only once has an exponent of 1.

Example 5

Rewrite 6^2 in expanded form.

$6 \cdot 6$

6 multiplied by itself 2 times.

Example 6

Rewrite $(-7)^4$ in expanded form.

$(-7)(-7)(-7)(-7)$

–7 multiplied by itself 4 times. Notice the negative must be used in the repeated multiplication.

Example 7

Rewrite $8a^3$ in expanded form.

$8 \cdot a \cdot a \cdot a$

8 times "a" multiplied by itself 3 times.

Example 8

Rewrite $-2^3 x^5$ in expanded form.

$-2 \cdot 2 \cdot 2 \cdot x \cdot x \cdot x \cdot x \cdot x$

*The negative is **not** used in the repeated multiplication.*

Example 9

Rewrite 9^1 in expanded form.

9

9 written 1 time.

An important rule for exponents to memorize is:

A base raised to the power of 0 will always equal 1.
(This rule will be discussed in more detail in chapter 9. For now, memorize it!)

For example:

$5^0 = 1$ $\quad x^0 = 1$ $\quad (-2)^0 = 1$ $\quad -8^0 = -1$ $\quad -3^0 x^2 = -1x^2 = -x^2$

Example 10

Find the value of 4^3.

$4^3 = 4 \cdot 4 \cdot 4$
$= 64$

4 multiplied by itself 3 times.

Example 11

Find the value of 3^0.

$3^0 = 1$

A base raised to the power of 0 will always equal 1.

Example 12

Find the value of 12^1.

$12^1 = 12$

12 written 1 time.

Example 13

Rewrite -3^4 in expanded form.

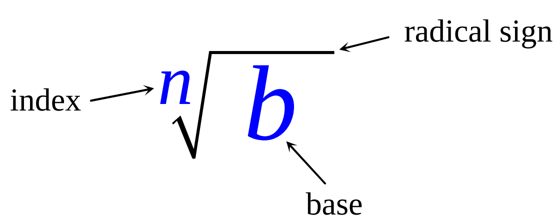
$-3^4 = -3 \times 3 \times 3 \times 3$
$= -81$

*The negative is **not** used in the repeated multiplication.*

Roots or Radicals

Finding a root of a number is the opposite operation of raising a number to a power or exponent. To learn the basics we will deal with roots of positive numbers only.

We will begin by looking at the notation used for roots.



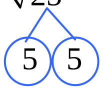
radical sign $\left(\sqrt{\ }\right)$ **–** tells us to find a root.

index (*n*) – tells us which root we are finding.
base (*b*) – the number to use to create a factor tree.

Note: $\sqrt{\ } = \sqrt[2]{\ }$ (If there is no index, it is the same as having an index of 2. This type of a root is known as a square root.)

To find a root, start by prime factoring the base. Look for groups of the same number. (The size of your group will depend on the index in your problem.) Once you have located a group of the same number, place that number on the outside of the radical. Study the following examples.

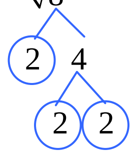
$\sqrt{25}$



There is no index, so we know we are looking for a square root or group of two. There are two 5's. The square root of 25 is 5.

$\sqrt{25} = 5$

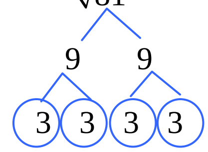
$\sqrt[3]{8}$



The index is a three. We are looking for groups of three. There are three 2's. the cube root of 8 is 2.

$\sqrt[3]{8} = 2$

$\sqrt[4]{81}$



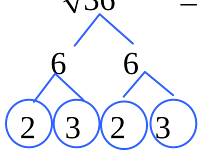
The index is four. We are looking for groups of four. There are four 3's. The fourth root of 81 is 3.

$\sqrt[4]{81} = 3$

Example 14

Find the indicated root. $\sqrt{36}$

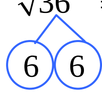
$\sqrt{36} \quad = \quad 2 \cdot 3 \quad = \quad 6$



We are looking for groups of two. There are two 2's and two 3's. The square root will be the product of 2 and 3, which is 6.

This problem can be done much quicker by identifying the two 6's.

$\sqrt{36} \quad = \quad 6$

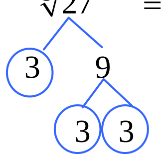


Identify a group of two.

Example 15

Find the indicated root. $\sqrt[3]{27}$

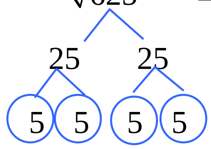
$\sqrt[3]{27} \quad = \quad 3$



We are looking for groups of three. There are three 3's. The cube root of 27 is 3.

Example 16 **Find the indicated root.** $\sqrt[4]{625}$

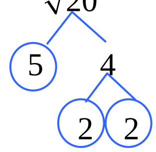
$$\sqrt[4]{625} = 5$$



We are looking for groups of four. There are four 5's. The forth root of 625 is 5.

What happens when you don't have equal groups? Study the following example.

$$\sqrt{20}$$

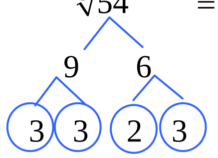


There is no index, so we are looking for groups of two. There is one group of 2's, but we have a 5 left over that is not in a group. When this happens, whatever is left without a group must stay under the radical sign.

$$\sqrt{20} = 2\sqrt{5}$$

Example 17 **Find the indicated root.** $\sqrt[3]{54}$

$$\sqrt[3]{54} = 3\sqrt[3]{2}$$



The index is 3, so we are looking for groups of three. There are three 3's and one two. We place a 3 outside the radical for the complete group. The 2 must stay under the radical.

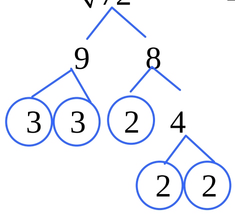
Example 18 **Find the indicated root.** $\sqrt{17}$

$$\sqrt{17}$$

17 is a prime number. There will not be any groups.

Example 19 **Find the indicated root.** $\sqrt{72}$

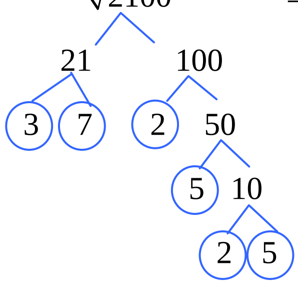
$$\sqrt{72} = 2\cdot 3\sqrt{2} = 6\sqrt{2}$$



We are looking for groups of two. There are two 3's and three 2's. place one 2 and one 3 on the outside. Leave one 2 inside the radical.

Example 20 **Find the indicated root.** $\sqrt{2100}$

$$\sqrt{2100} = 5\cdot 2\sqrt{3\cdot 7} = 10\sqrt{21}$$



We are looking for groups of two. There are two 3's and three 2's. Place one 2 and one 3 on the outside. Leave one 2 inside the radical.

The following table of perfect squares and perfect cubes may help you with roots. This table is also located in the Important Information Section of your book.

SQUARES	SQUARE ROOTS	CUBES
$1^2 = 1$	$\sqrt{1} = 1$	$1^3 = 1$
$2^2 = 4$	$\sqrt{4} = 2$	$2^3 = 8$
$3^2 = 9$	$\sqrt{9} = 3$	$3^3 = 27$
$4^2 = 16$	$\sqrt{16} = 4$	$4^3 = 64$
$5^2 = 25$	$\sqrt{25} = 5$	$5^3 = 125$
$6^2 = 36$	$\sqrt{36} = 6$	$6^3 = 216$
$7^2 = 49$	$\sqrt{49} = 7$	$7^3 = 343$
$8^2 = 64$	$\sqrt{64} = 8$	$8^3 = 512$
$9^2 = 81$	$\sqrt{81} = 9$	$9^3 = 729$
$10^2 = 100$	$\sqrt{100} = 10$	$10^3 = 1000$
$11^2 = 121$	$\sqrt{121} = 11$	$11^3 = 1331$
$12^2 = 144$	$\sqrt{144} = 12$	$12^3 = 1728$

2.2 EXERCISES

In 1-5, rewrite the following with an exponent.

1. $2 \cdot 2 \cdot 2 \cdot a \cdot a$
2. $(-6)(-6)(-6)$
3. $-4 \cdot 4 \cdot 4 \cdot 4 \cdot 4$
4. $3 \cdot x \cdot x \cdot y \cdot z \cdot z \cdot z$
5. $(x)(x)$

In 6-10, rewrite the following in expanded form.

6. p^6
7. $(-5)^4$
8. -5^4
9. $-9x^3$
10. $7x^2y^5$

In 11-15, find the value of the following.

11. -2^6
12. 1248^0
13. 11^2
14. $(-3)^4$
15. $-7^0x^0y^2$

In 16-23, find the indicated root.

16. $\sqrt{49}$
17. $\sqrt{124}$
18. $\sqrt[3]{125}$
19. $\sqrt[4]{162}$

20. $\sqrt{99}$

21. $\sqrt[5]{300,000}$

22. $\sqrt[3]{40}$

23. $\sqrt{252}$

Applications.

24. In 1990, Sanpete County ranked 1st in the United States in total population of turkeys with more than 9^8 turkeys. What was the approximate turkey population of Sanpete County?

25. A new fast food restaurant opened in a small town in Southern Utah. During its first week of business the owner projected the number of customers to be served would be 15^3. At the end of the first week the owner calculated that they actually served 8^4 customers. How many customers were served above the owners projected amount?

2.3 Order of Operations & Evaluating Algebraic Expressions

Order of Operations

We have discussed the mathematical operations of addition, subtraction, multiplication, division, evaluating exponents, and finding roots. The majority of the problems to this point involved only one of these operations. What happens when you have a problem that contains more than one of these operations? Which operation do you perform first? One might have the tendency to perform the operations in order from left to right, because that is how we read. This is not necessarily the case. Not all languages are read from left to right, some are read top to bottom. Mathematics is a universal language. Because of this, the **Order of Operations** were established. The Order of Operations guarantee that the steps will be done in the same order regardless the language you speak or the country you are in. The following is a table that outlines the steps to the Order of Operations.

> **Order of Operations**
>
> 1. Simplify within **parentheses** ( ) and other grouping symbols, such as **brackets** [ ], **braces** { }, or the **fraction bar —**. *(When more than one pair of grouping symbols occur within a problem, work the innermost set of grouping symbols first.)*
> 2. Evaluate an expression involving **exponents** or **roots**.
> 3. **Multiply** or **divide** in order from left to right.
> 4. **Add** or **subtract** in order from left to right.

When following the steps in the Order of Operations, you must complete each step, if applicable, before going on to the next. It is important to note in steps 3 and 4, multiplication does not take precedence over division and addition does not take precedence over subtraction. Perform these operations in order from left to right.

Example 1 **Simplify** $5+2\cdot 8$

$5+\underbrace{2\cdot 8}$ *There are no parentheses, exponents or roots, so we will begin with multiplication.*

$\underbrace{5+16}$ *Add.*

21

Example 2 **Simplify** $24-(5-2)\cdot 6$

$24-(\underbrace{5-2})\cdot 6$ *Start by simplifying within the parentheses.*

$24-\underbrace{3\cdot 6}$ *Multiply.*

$\underbrace{24-18}$ *Subtract.*

6

Example 3

Simplify $(9-8)^3+3\cdot 2^4$

$(9-8)^3+3\cdot 2^4$	*Start by simplifying within the parentheses.*
$(1)^3+3\cdot 2^4$	*Evaluate the exponents.*
$1+3\cdot 16$	*Multiply.*
$1+48$	*Add.*
49	

Example 4

Simplify $75\div 5\cdot 3-30+3$

$75\div 5\cdot 3-30+3$	*There are no parentheses, exponents, or roots. Begin with multiplication and division. Perform these operations in order from left to right. Because the operation of division is on the left, do this first.*
$15\cdot 3-30+3$	*Multiply.*
$45-30+3$	*Add or subtract in order from left to right.*
$15+3$	
18	

Example 5

Simplify $5\cdot 9-(4+8)\div 2-(7\times 4)$

$5\cdot 9-(4+8)\div 2-(7\times 4)$	*Start by simplifying within the parentheses.*
$5\cdot 9-12\div 2-28$	*Multiply.*
$45-12\div 2-28$	*Divide.*
$45-6-28$	*Subtract. Work left to right.*
$39-28$	*Subtract.*
11	

Example 6

Simplify $\dfrac{60-5^2+1}{-3(1+1)}$

$\dfrac{60-5^2+1}{-3(1+1)}$ *The fraction bar serves as a grouping symbol. Completely simplify the top and completely simplify the bottom before you divide. Start by simplify within the parentheses on the bottom.*

$\dfrac{60-5^2+1}{-3(2)}$ *Evaluate the exponent.*

$\dfrac{60-25+1}{-3(2)}$ *Multiply.*

$\dfrac{60-25+1}{-6}$ *Add or subtract in order from left to right.*

$\dfrac{35+1}{-6}$

$\dfrac{36}{-6}$ *Divide.*

-6

Example 7

Simplify $7\left[21-8\left(\sqrt{25}-3\right)\right]$

$7\left[21-8\left(\sqrt{25}-3\right)\right]$ *Start by simplifying within the innermost parentheses. Find the value of the root.*

$7\left[21-8(5-3)\right]$ *Subtract within the innermost parentheses.*

$7\left[21-8(2)\right]$ *Multiply*

$7[21-16]$ *Subtract.*

$7[5]$ *Multiply.*

35

Example 8

Simplify $1+5\{3-2[4^2-3(2+1)]\}$

$1+5\{3-2[4^2-3(2+1)]\}$ *Start by simplifying within the innermost parentheses. Add.*

$1+5\{3-2[4^2-3(3)]\}$ *Simplify within the brackets. Evaluate the exponent.*

$1+5\{3-2[16-3(3)]\}$ *Multiply within the brackets.*

$1+5\{3-2[16-9]\}$ *Subtract within the brackets.*

$1+5\{3-2[7]\}$ *Multiply within the braces.*

$1+5\{3-14\}$ *Subtract within the braces.*

$1+5\{-11\}$ *Multiply.*

$1+-55$ *Add.*

-54

Average

A common application problem for the Order of Operation is finding **average**. To find the average, find the sum of your values, then divide that sum by the total number of values. For example, Beth's test scores are 97%, 81%, 91%, and 71%. Find her average test score.

$$\frac{97+81+91+71}{4} = \frac{340}{4} = 85\%$$

Example 9

Find the average of 1, 9, –20, and –6.

$\frac{1+9+-20+-6}{4}$ *Find the sum of all the values and divide that sum by the number of values.*

$\frac{-16}{4}$ *Divide.*

-4

Example 10 **Find the average of 7, 9, 0, 15, –3, –8, and –6.**

$$\frac{7+9+0+15+(-3)+(-8)+(-6)}{7}$$ *Find the sum of all the values and divide that sum by the number of values.*

$$\frac{14}{7}$$ *Divide.*

$$2$$

Example 11 **Brian, the center for the Badgers basketball team, scored 25 points, 15 points, 10 points, 23 points, and 45 points in the last five basketball games. Find his average points per game.**

$$\frac{25+15+10+23+45}{5}$$ *Find the sum of all the values and divide that sum by the number of values.*

$$\frac{118}{5}$$ *Divide.*

$$23.6$$

Brian averaged 23.6 points per game.

EVALUATING ALGEBRAIC EXPRESSIONS

We will now apply the Order of Operations when simplifying algebraic expressions.

An **algebraic expression** is any single variable or number or any grouping of variables and numbers without an equal sign. Examples of algebraic expressions include:

$$5x, \quad -2y+7, \quad 4xz^2-6y+8, \quad \frac{4a+2b^3}{5c}$$

To simplify an algebraic expression, replace the variable(s) with their given numerical value(s). Follow the Order of Operations to simplify the expression. Finding the value of the expression is also called **evaluating the expression** for the variable.

Example 12

Evaluate $x + 3$ if $x = 4$.

$$x + 3 = 4 + 3$$
$$= 7$$

Replace x with 4. Add.

Example 13

Evaluate $2(r - 5)$ if $r = 3$.

$$2(r-5) = 2(3-5)$$

Replace r with 3. Simplify within the parentheses by subtracting.

$$= 2(-2)$$

Multiply.

$$= -4$$

Example 14

Evaluate $2(x + y)$ if $x = 4$ and $y = -1$.

$$2(x+y) = 2(4+-1)$$

Replace x with 4 and y with –1. Simplify within the parentheses by adding.

$$= 2(3)$$

Multiply.

$$= 6$$

Example 15

Evaluate $\frac{b+6}{a}$ if $a = 2$ and $b = 8$.

Replace a with 2 and b with 8.

$$\frac{b+6}{a} = \frac{8+6}{2}$$

The fraction bar serves as a grouping symbol. Completely simplify the top and completely simplify the bottom before dividing. Start by adding on the top.

$$= \frac{14}{2}$$

Divide.

$$= 7$$

Example 16

Evaluate $\sqrt{r}-3s^2$ if r = 16 and s = –5.

$$\sqrt{r}-3s^2=\sqrt{16}-3(-5)^2$$
Replace r with 16 and x with –5. Evaluate the root and exponent.

$$=4-3(25)$$
Multiply.

$$=4-75$$
Subtract.

$$=-71$$

Example 17

Evaluate $\dfrac{x^2+z-3}{y}$ if x = 5, y = 2 and z = 4.

Replace x with 5, y with 2, and z with 4.

$$\frac{x^2+z-3}{y}=\frac{(5)^2+4-3}{2}$$

The fraction bar serves as a grouping symbol. Completely simplify the top and completely simplify the bottom before dividing. Start by evaluating the exponent.

$$=\frac{25+4-3}{2}$$
Add or subtract in order from left to right.

$$=\frac{26}{2}$$
Divide.

$$=13$$

In application problems you may find it necessary to first write an algebraic expression before you can evaluate the expression. Recall from chapter 1 the words that represent the different mathematical operations.

Example 18

Write an algebraic expression for each of the following. Use x to represent "*a number*". Evaluate the expression when x = 2.

a) Twice a number

Words:	*Twice*		*a number*
Algebraic Expression:	2	$\cdot$	x

Evaluate:

$$2\cdot x=2\cdot 2$$
$$=4$$

b) Negative ten minus a number

Words:	*Negative ten*	*minus*	*a number*
Algebraic Expression:	-10	$-$	x

Evaluate:

$$-10 - x = -10 - 2$$
$$= -12$$

c) Seven more than a number *(the word* ***than*** *will reverse the order)*

Words:	*seven*	*more than*	*a number*
Expression:	x	$+$	7

Evaluate:

$$x + 7 = 2 + 7$$
$$= 9$$

d) The product of a number and negative three

The product of

Words:	*a number*	*and*	*negative three*
Expression:	x	$\cdot$	(-3)

Evaluate:

$$x \cdot (-3) = 2 \cdot (-3)$$
$$= -6$$

e) Eight times a number divided by four.

Words:	*eight*	*times*	*a number*	*divided by*	*four*
Expression:	8	$\cdot$	x	$\div$	4

Evaluate:

$$8 \cdot x \div 4 = 8 \cdot 2 \div 4$$
$$= 16 \div 4$$
$$= 4$$

2.3 EXERCISES

In 1-20, use the order of operations to simplify the following expressions.

1. $5+2\cdot 3$
2. $24\div 3-10$
3. $72\div 4\cdot 2$
4. $8-3\cdot 5$
5. $4\times 2+9\times 3$
6. $-4+8\div 2$
7. $12+\dfrac{18}{(-3)}$
8. $(4+5)\div 3$
9. $\dfrac{18+6}{4-2^4}$
10. $\sqrt{81}+5-3^2$
11. $6^2\cdot(10-8)$
12. $\dfrac{-43-17}{3^2-4}$
13. $(5^2+5)\div 5$
14. $5\div 0+18$
15. $3^4-[35-(12-6)]$
16. $\dfrac{5(12-7)-4}{5^2-2^3-10}$
17. $(7\cdot 5)+[9\div(3\div 3)]$
18. $5+4[2-3(12\div 3+1)]$
19. $4+\{2-5[3^2+4(\sqrt{16}-3)]\}$
20. $29-\{5+3[8\cdot(\sqrt{100}-8)]-50\}$

In 21-25, find the average.

21. Find the average of 12, 7, 0, 8, & 23.

22. Find the average of –5, –10, 18, & –27.

23. Tommy's allowance for the last 6 weeks was \$10, \$12, \$15, \$10, \$8, and \$11. What was his average weekly allowance?

24. Cali's test scores in her English class were 84%, 73%, 98% and 90%. What was her average test score.

25. The morning temperatures for the last week were 7^{o}, 9^{o}, 4^{o}, 0^{o}, -3^{o}, 5^{o}, and -1^{o}. What was the average morning temperature?

In 26-33, evaluate the following expressions for x = 5, y = –2, and z = 3.

26. $4+5x$

27. $2x+y$

28. $-3x-2y+z$

29. $2xy^{2}-5$

30. $\dfrac{3x-z}{y}$

31. $\left(2x-y\right)^{2}$

32. $xy\left(2x-3y+z\right)$

33. $\dfrac{y^{3}+z^{2}+18}{x}$

In 34-41, write an algebraic expression for the each of the following. Use x, to represent "a number". Evaluate the expression when x = 2.

34. The sum of three and a number.

35. The difference between a number and twelve.

36. The quotient of six and a number.

37. The product of a number and negative seven.

38. Fifteen less than a number.

39. Twenty-four decreased by twice a number.

40. The square of a number increased by negative seven.

41. The difference of three times a number and four.

2.4 REVIEW EXERCISES

In 1-4, use a factor tree to write the prime factorization of each number.

1. 30
2. 79
3. 720
4. 1188

In 5-10, find the greatest common factor (GCF) of the following.

5. 6 and 15
6. 18 and 35
7. -21 and 14
8. $12, -6$, and 3
9. 27, 36, and 54
10. -18, 9, and 36

In 11-14, find the least common multiple (LCM) of the following.

11. 5 and 30
12. 12 and 40
13. 15, 45, and 100
14. 6, 9, and 27

In 15-18, rewrite the following with an exponent.

15. $4 \cdot 4 \cdot 4 \cdot 4 \cdot 4$
16. $(-2)(-2)(-2)x \cdot x \cdot x \cdot x$
17. $-1 \cdot 1 \cdot 1$
18. $5 \cdot 5 \cdot 5 \cdot a \cdot a \cdot a \cdot a \cdot b \cdot b \cdot c$

In 19-22, rewrite the following in expanded form.

19. x^3
20. -5^4
21. $2^3 x^2 y^3$
22. $-10x^4 yz^2$

In 23-26, find the value of the following.

23. 2^8
24. $(-4)^2$
25. -4^2
26. 211^0

In 27-30, find the indicated root.

27. $\sqrt{100}$

28. $\sqrt[3]{1080}$

29. $\sqrt[4]{625}$

30. $\sqrt[5]{320}$

In 31-36, use the order of operations to simplify the following expressions.

31. $6 \div 3 + 5^2$

32. $2^5 \div 4 \cdot 2 + \sqrt{9}$

33. $2 + 6\left[4^2 - (2-4)\right]$

34. $\dfrac{4(3+2)+4}{-4-2}$

35. $7\left[3 - 5(4-6)\right]$

36. $7^2 - \left\{18 - \left[40 \div (4 \cdot 2) + 2\right] + 5^2\right\}$

In 37-38, find the average.

37. Find the average of 12, 0, –1, –8, 10, 3, –7, and 15.

38. Ben earned 75%, 84% and 93% on his Biology exams. What is his average test score?

In 39-42, evaluate the following expressions for x = 2, y = –3, and z = 5.

39. $5 + 4x$

40. $x^2 - y$

41. $3x + 2y - 4z$

42. $\dfrac{7y^2 + 2z - 1}{x}$

In 43-46, write an algebraic expression for the each of the following. Use x, to represent "a number". Evaluate the expression when x = 2.

43. The sum of a number and (–5).

44. 4 less than a number.

45. The quotient of (–8) and a number.

46. Twice a number increased by 3.