1.1 The Number System and Its Properties

To begin understanding mathematics and its properties we must first identify the different groups or sets of numbers.

NATURAL NUMBERS (N)

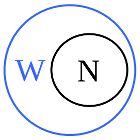
The set of natural numbers are known as the counting numbers. They include 1, 2, 3, 4, 5, … The set of natural numbers is written:

$$N = \{1, 2, 3, 4, 5, \ldots\}$$

WHOLE NUMBERS (W)

The set of whole numbers include all the natural numbers and zero. The set of whole numbers is written:

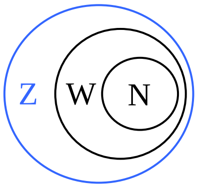
$$W = \{0, 1, 2, 3, 4, 5, \ldots\}$$



INTEGERS (Z)

The set of integers include all the whole numbers and their opposites. The set of integers is written:

$$Z = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$$



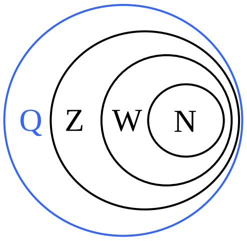
RATIONAL NUMBERS (Q)

The set of rational numbers include all the integers along with any number that can be expressed as a ratio or fraction $\frac{a}{b}$, where a and b are integers and $b \neq 0$. The decimal form of a rational number is either a terminating or a repeating decimal.

Examples:

$$\frac{5}{17},\ \frac{1}{2},\ 1.62626262\ldots\ \text{(repeating)},$$

$$-7.34\ \text{(terminating)},\ 0,\ \text{etc.}$$



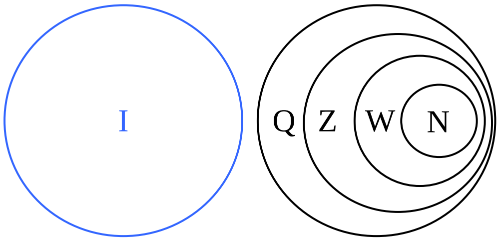
IRRATIONAL NUMBERS (I)

An irrational number is any number that is not a rational number. In other words, irrational numbers are numbers that cannot be written as a ratio or fraction. The decimal form of an irrational number is a non-terminating and non-repeating decimal.

Examples:

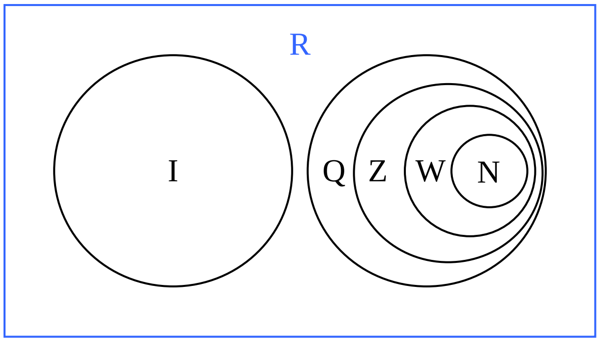
$\pi = 3.14159265359...,\ \sqrt{7},$

$-\sqrt{2},\ e = 2.71828182846...,$ etc.



REAL NUMBERS (R)

Real numbers include all the sets of numbers previously discussed. Real numbers can be thought of as the numbers we use in everyday life.



PROPERTIES OF REAL NUMBERS

The numbers we have discussed have actions or operations they can do. These are referred to as Properties. You will be familiar with many of these properties as you use them frequently in arithmetic problems. It is important now for you to learn and use their proper names.

COMMUTATIVE PROPERTY OF ADDITION

For all real numbers *a* and *b*,

$$a + b = b + a$$

When you add two values, the order in which you add does not matter. For example:

$$2 + 3 = 3 + 2$$

$$5 = 5$$

COMMUTATIVE PROPERTY OF MULTIPLICATION

For all real numbers *a* and *b,*

$$a \cdot b = b \cdot a$$

When you multiply two values, the order multiply does not matter. For example:

$$4 \cdot 5 = 5 \cdot 4$$

$$20 = 20$$

Note: There is not a commutative property for subtraction and division.

ASSOCIATIVE PROPERTY OF ADDITION

For all real numbers *a, b,* and *c,*

$$a + (b + c) = (a + b) + c$$

The associative property of addition is similar to the commutative property of addition. The only difference is that the associative property holds true for more than two values. For example:

$$1 + (4 + 7) = (1 + 4) + 7$$

$$1 + 11 = 5 + 7$$

$$12 = 12$$

ASSOCIATIVE PROPERTY OF MULTIPLICATION

For all real numbers *a, b,* and *c,*

$$a \cdot (b \cdot c) = (a \cdot b) \cdot c$$

The associative property of multiplication is similar to the commutative property of multiplication. The only difference is that the associative property holds true for more than two values. For example:

$$3 \cdot (2 \cdot 5) = (3 \cdot 2) \cdot 5$$

$$3 \cdot 10 = 6 \cdot 5$$

$$30 = 30$$

Note: There is not an associative property for subtraction and division.

The following four properties are referred to as identity properties. This name "identity" comes from the fact that when we perform an operation on a number, the number does not change, it holds its identity.

IDENTITY PROPERTY OF ADDITION

For all real numbers *a*,

$$a + 0 = a$$

When you add zero to a number, you get that number. For example:

$$7 + 0 = 7$$

IDENTITY PROPERTY OF SUBTRACTION

For all real numbers *a*,

$$a - 0 = a$$

When you subtract zero from a number, you get that number. For example:

$$5 - 0 = 5$$

IDENTITY PROPERTY OF MULTIPLICATION

For all real numbers *a*,

$$a \cdot 1 = a$$

When you multiply a number by one, you get that number. For example:

$$3 \cdot 1 = 3$$

IDENTITY PROPERTY OF DIVISION

For all real numbers *a*,

$$a \div 1 = \frac{a}{1} = a$$

When you divide a number by one, you get that number. For example:

$$8 \div 1 = 8 \qquad \text{and} \qquad \frac{15}{1} = 15$$

MULTIPLICATION PROPERTY OF ZERO

For all real numbers *a*,

$$a \cdot 0 = 0$$

A number multiplied by zero will always give you a product of zero. For example:

$$2547 \cdot 0 = 0$$

DIVISION INTO ZERO PROPERTY

For all real numbers *a*, $a \neq 0$

$$0 \div a = \frac{0}{a} = 0$$

Zero divided by a non-zero number will equal zero. For example:

$$0 \div (-27) = 0 \qquad \text{and} \qquad \frac{0}{13} = 0$$

To understand this property we can think of division in terms of multiplication. In our examples, ask the following questions:

“What number times $-27 = 0$?”

“What number times 13 = 0 ?”

The answer to these questions is zero: $-27 \cdot 0 = 0, \quad 13 \cdot 0 = 0$. Therefore, whenever you divide zero by a non-zero number, you will always get zero.

DIVISION BY ZERO PROPERTY

For all non-zero real numbers *a*,

$$a \div 0 = \frac{a}{0} = \text{undefined}$$

Division by zero is always undefined. For example:

$$17 \div 0 = \text{undefined} \qquad \text{and} \qquad \frac{-12}{0} = \text{undefined}$$

To understand this property we can again think of division in terms of multiplication. In our examples, ask the following questions:

“What number times $0 = 17$?”

“What number times $0 = -12$?”

The answer is that you cannot multiply zero by any number to get anything other than zero. Therefore, division by zero cannot be done and is said to be **undefined**.

1.1 EXERCISES

In 1-10, determine whether the following statements are <u>true</u> or <u>false</u>.

1. All integers are whole numbers.
2. 0 is a natural number.
3. All rational numbers can be written as a ratio or a fraction.
4. $\sqrt{11}$ is a rational number.
5. All natural numbers are rational numbers.
6. All irrational numbers are real numbers.
7. -7 is an integer and a whole number.
8. Real numbers are numbers we use in everyday life.
9. All negative numbers are integers.
10. $\frac{1}{2}$ is a rational number and a real number.

In 11-15, read and respond to the following exercises.

11. Which set(s) of numbers contains the number 4?
12. What letter is used to represent the set of integers? The set of rational numbers?
13. In your own words, define the set of rational numbers.
14. What is the difference between the set of integers and the set of whole numbers? What are their similarities?
15. Give one example of a number that belongs to the following sets of numbers:
    a. Natural Numbers
    b. Whole Numbers
    c. Integers
    d. Rational Numbers
    e. Irrational Numbers
    f. Real Numbers

In 16-25, Match each statement with the property that it illustrates. You may need to use a property more than once.

A. Commutative Property of Addition
B. Commutative Property of Multiplication
C. Associative Property of Addition
D. Associative Property of Multiplication
E. Identity Property of Addition
F. Identity Property of Subtraction
G. Identity Property of Multiplication
H. Identity Property of Division
I. Multiplicative Property of Zero
J. Division into Zero Property
K. Division by Zero Property

16. $7 \cdot 1 = 1 \cdot 7$

17. $9 \div 0 = undefined$

18. $2 + (4 + 5) = (2 + 4) + 5$

19. $9 \cdot 1 = 9$

20. $15 + 0 = 0 + 15$

21. $5 \cdot (2 \cdot 3) = (5 \cdot 2) \cdot 3$

22. $\frac{0}{3} = 0$

23. $(1 + 8) + 3 = 1 + (8 + 3)$

24. $0 = 21 \cdot 0$

25. $3 \cdot 9 = 9 \cdot 3$

26. $4 + 0 = 0$

27. $(4 \cdot 1) \cdot 7 = 4 \cdot (1 \cdot 7)$

28. $14 \div 1 = 14$

29. $3 + 6 = 6 + 3$

30. $10 + (2 + 3) = (10 + 2) + 3$

31. $0 \div 11 = 0$

32. $10 \cdot 2 = 2 \cdot 10$

33. $15 \cdot 0 = 0$

34. $\frac{3}{0} = undefined$

35. $19 = 19 - 0$

In 36-38, read and respond to the following exercises.

36. Describe and give an example of why subtraction is not commutative.

37. In your own words, explain why division by zero is undefined.

38. Using the following example, show why division is not associative.

$$24 \div (4 \div 2) = (24 \div 4) \div 2$$

1.2 Place Value

Every number has a value that is determined by the place it is located. Think about money. For example, if you have the number 2 and it is located before the decimal point, 2.00, you have two dollars. If the two is located one space behind the decimal, 0.20, you have twenty cents. If the two is located two spaces behind the decimal, 0.02, you have two cents.

In our number system, each number has a specific place value. Figure 1.1 (below), illustrates this concept.

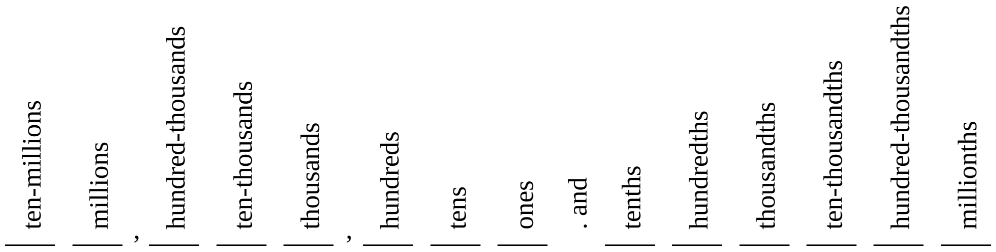


Figure 1.1

It is important to be able to identify the place value a number holds. The numbers located before the decimal point are referred to as integers. (You will notice there is no “**th**” in their name.) The numbers after the decimal point represent a part of an integer, also referred to as a real number. (You will notice there is a **“th”** at the end of their name.)

Integers use **commas** in both word form and numerical form. Commas are placed after each group of three digits. (Notice where the commas fall in Figure 1.1). Examine the following numbers for the correct location of the commas:

2,180	*Two thousand, one hundred eighty*
1,467,210	*One million, four hundred sixty-seven, two hundred ten*

When writing numbers, you need to know which numbers are hyphenated. The numbers between twenty and one hundred that are read as a group of two numbers and will include a hyphen. For instance:

37	*thirty-seven*
149	*one hundred forty-nine*

If there is no sign in front of your number, it is positive. If the sign (–) is in front of your number, it is negative and needs to be read as such.

Example 1

Write the Integers in words.

a) **2,179,536**

Two million, one hundred seventy-nine thousand, five hundred thirty-six.

b) **–460,254**

Negative four hundred sixty thousand, two hundred fifty-four.

Notice the commas are in the same location in both the numerical form and the written form. Also notice the location of the hyphens.

Example 2

Write the Integers in digits.

a) **Negative fifty-four thousand, seven hundred eight.**

–54,708

b) **Two hundred three million, fourteen thousand, eighty-five.**

203,014,085

When reading Real Numbers, it is important to note that the **decimal point** is read "**and**". The numbers to the left of the decimal point are read as integers. The numbers to the right of the decimal point are read like an integer, but end with the place value name of the last digit. If there is no sign in front of your number, it is positive. If the sign (–) is in front of your number, it is negative and needs to be read as such. For example, the number –1.34 is read *negative one and thirty-four hundredths.*

Example 3

Write the number 23.2 in words.

Twenty-three and two tenths

Example 4

Write the number 0.715 in words.

Seven hundred fifteen thousandths

It is not necessary to say "zero and seven hundred fifteen thousandths." Numbers that only have digits to the right of the decimal do not require the "and" for the decimal point.

Example 5

Write the number –475,836.02 in words.

Negative four hundred seventy-five thousand, eight hundred thirty-six and two hundredths

Writing numbers is very similar to reading numbers. Start by looking for the decimal place, the "and." The numbers to the left of the "and" are the integers. The numbers to the right of the "and" are the decimals. The last word in the number will identify the number of decimal places needed. When a place value does not have a digit assigned to it, it is represented with a zero.

For example: **six and seventy-four thousandths**, six is the whole number, a decimal point for the "and", followed by three decimal places (thousandths).

6. ____ ____ ____

Fill in the decimal places with their appropriate numbers.

6. ___ _7_ _4_

Use a zero to fill in the unassigned place value.

6. _0_ _7_ _4_

6.074

Example 6

Write seven and four hundredths in digits.

____. ____ ____

We need one digit to the left of the decimal point because we have a number that only occupies the ones place value. We need two digits to the right of the decimal point for the hundredths.

_7_. ____ ____

Fill in the integer, 7.

_7_. _**0**_ _4_

Fill in the decimals, including any 0's for the digits that do not have values assigned to them.

7.04

Example 7

Write three thousand, four hundred twenty-seven and four thousand five hundred eighty-nine ten-thousandths in digits.

____, ____ ____ ____. ____ ____ ____ ____

We need four digits to the left of the decimal point because we read thousand. We need three digits to the right of the decimal point for the ten-thousandths.

_3_, _4_ _2_ _7_. ____ ____ ____ ____

Fill in the whole integer, 3427.

_3_, _4_ _2_ _7_. _4_ _5_ _8_ _9_

Fill in the decimals, including any 0's for the digits that do not have values assigned to them.

3427.4589

Example 8 **Write negative six thousand three millionths in digits.**

– ____. ____ ____ ____ ____ ____ ____

Start with a negative sign. We need one digit to the left of the decimal point for the integer and six digits to the right of the decimal point for the millionths.

–_0_. ____ ____ ____ ____ ____ ____

There is no integer stated, so we will represent our integer with a 0.

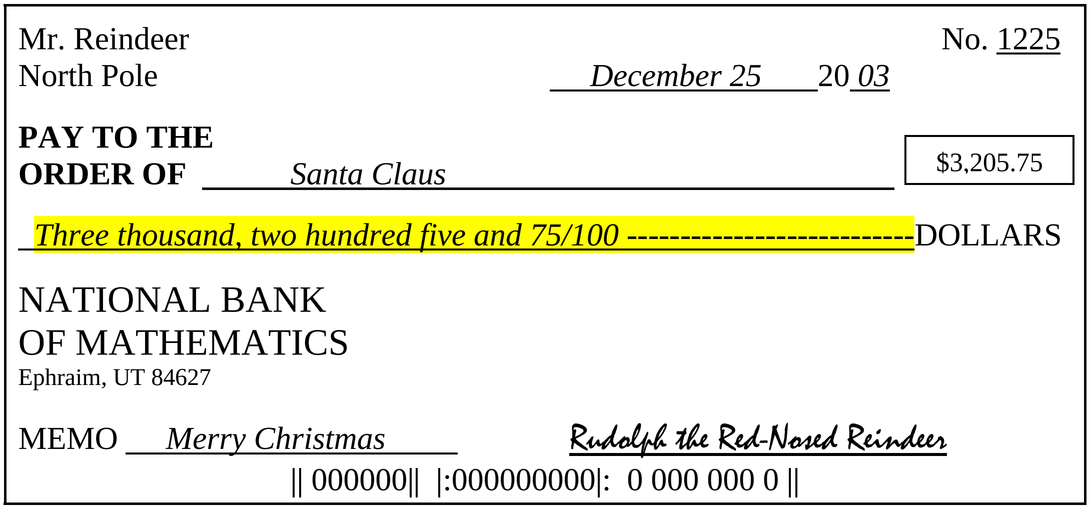
–_0_. _0_ _0_ _6_ _0_ _0_ _3_

Fill in the decimals, including any 0's for the digits that do not have values assigned to them.

–0.006003

We can apply the concept of changing words to digits and digits to words to the everyday life task of writing checks. When writing a check, the numbers to the left of the decimal place, the dollars, will be written in words, followed by an "**and**" for the decimal. The numbers to the right of the decimal place are the cents. The cents will be left as digits and written over 100. Example 9 properly illustrates how to write a check.

Example 9 **Fill in the amount of the check.**



Mr. Reindeer
North Pole

No. 1225

December 25 20 *03*

PAY TO THE
ORDER OF *Santa Claus*

$3,205.75

*Three thousand, two hundred five and 75/100 ---------------------------*DOLLARS

NATIONAL BANK
OF MATHEMATICS
Ephraim, UT 84627

MEMO *Merry Christmas*

Rudolph the Red-Nosed Reindeer

|| 000000|| |:000000000|: 0 000 000 0 ||

While we are studying place value, it is good to think about where the numbers lie on a number line and how numbers compare one with another. In Figure 1.2 (below), you will find a number line. The numbers to the left are always less than the numbers to the right.

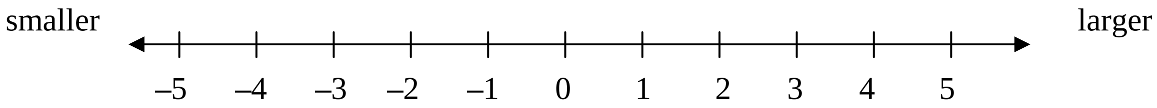
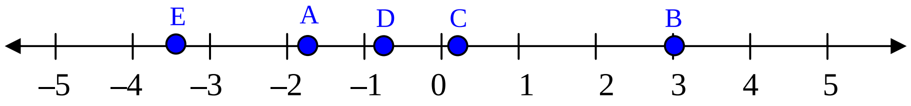


Figure 1.2

Example 10

Plot and label the following numbers on the number line below.

A. –1.6 **B. 3** **C. 0.25** **D. –0.715** **E. –3.4**



When comparing numbers we will use the following symbols, $>$, $<$, and $=$. The symbol $>$ means "is greater than," the symbol $<$ means "is less than," and the symbol = means "is equal to." For example:

The statement *2 is less than four* can be written with symbols as:

$$2 < 4$$

Also, the statement *–3 is greater than –7* can be written as:

$$-3 > -7$$

Finally, the statement *5 is equal to 5* can be written as:

$$5 = 5$$

Example 11

Compare the numbers –2.4 and 2.4. Use $>$, $<$, or =.

$$2.4 > -2.4$$

2.4 is greater than –2.4, because it is to the right of –2.4 on a number line. It is important to note that a positive number is always larger than a negative number.

Example 12

Compare the numbers -2.46 and –3.02. Use $>$, $<$, or =.

$$-3.02 < -2.46$$

–3.02 is less than –2.46 because it is to the left of –2.46 on a number line.

1.2 EXERCISES

In 1-6, write these numbers in words.

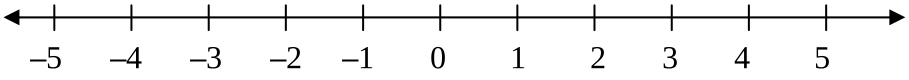
1. 17.1
2. 0.416
3. –34.58
4. 4,785.329
5. –9,472.20073
6. 600.0005

In 7-11, change from words into digits.

7. Six and three tenths
8. Negative one hundred twenty-six and seven thousandths
9. Four thousand, two hundred five and six hundredths
10. Three hundred eighty-nine millionths
11. Negative seven million, six hundred thousand, forty-seven and five hundred thirty-nine ten-thousandths

In 12-17, plot and label the following numbers on the number line.

12. (A) 2.5
13. (B) –1.75
14. (C) 4.3
15. (D) –0.2
16. (E) –5
17. (F) 3



In 18-23, compare the following numbers. **Use** >, <, or =.

18. –1.347 _____ 1.347
19. 1.72 _____ 0.43
20. –2.68 _____ –3.14
21. 8.0024 _____ 8.0025
22. 3.68 _____ –4.53
23. –12.35 _____ –12.39

In 24-28, solve the following application problems.

24. Fill in the amount of the check.

Mr. Easter Bunny
Tulip Field, USA
No. 411
April 11 20*04*

PAY TO THE
ORDER OF *Colored Eggs Express* $12,637.21

____ DOLLARS

NATIONAL BANK
OF MATHEMATICS
Ephraim, UT 84627

MEMO *5 Million Dozens of Eggs* Easter Bunny

|| 000000|| |:000000000|: 0 000 000 0 ||

25. Fill in the amount of the check.

Mrs. Tooth Fairy
100 Toothless Avenue
Denture, USA
No. 000
June 1 20*04*

PAY TO THE
ORDER OF *Toothless Grandma* $7,809.34

____ DOLLARS

NATIONAL BANK
OF MATHEMATICS
Ephraim, UT 84627

MEMO *$$ to help pay for those dentures.* Tooth Fairy

|| 000000|| |:000000000|: 0 000 000 0 ||

26. Gus and Todd work for a Construction Company. Gus earns one thousand, six hundred sixteen dollars and twelve cents per month. Todd earns one thousand, five hundred seventy-nine dollars and twenty-five cents per month. Convert their pay into digits. Who makes more money per month?

27. Bradley lost his wallet. In it he had one hundred twenty-seven dollars and thirty-six cents. Scott also lost his wallet with one hundred thirty-six dollars and forty-five cents. Convert the amount each lost into digits. Who lost the least amount of money?

28. Come up with a number that holds at least seven place values. Write the number in words and in digits.

1.3 Rounding

In Section 1.2 we learned about place value. In this section we will use place value to help us with the concept of rounding. We round numbers everyday without knowing we are doing it. For example, if we want to buy an item at the store that has a price tag of $4.99, we say that the item costs $5.00. This is an example of rounding to the nearest dollar.

In figure 1.3 (below), you will find rules to help you when rounding integers.

1. Locate the place value being considered, circle it.
2. Look at the number directly right of the circled number:
   A. If it is less than five, the circled number stays the same and the digits that follow change to zeros.
   B. If it is greater than or equal to five, add one to the circled number and the digits that follow will change to zeros.
   (Note: when a 9 is the number in question, and it needs to be increased, the change also involves the digit to the left of the 9.)

Figure 1.3

Example 1

Round the following to the nearest <u>hundreds</u>.

a. 12,467

12,④67

4 is the digit in the hundreds place. Circle it.

12,500

The number to the right is a 6. Because 6 is greater than 5, increase the 4 by one and replace the digits that follow with zeros.

b. 14

⓪14

There is not a digit in the hundreds place. When this occurs, we consider that number to be a 0.

000

The number to the right of the place value is a 1. Because 1 is less than 5, leave the 0 and replace the digits that follow with 0's.

0

Example 2

Round the following to the nearest <u>thousands</u>.

a. 157,243

15⑦,243

7 is the digit in the thousands place. Circle it.

157,000

The number to the right is a 2. Because 2 is less than 5, leave the 7 and replace the digits that follow with 0's.

b. 19,860

1⑨,860

9 is the digit in the thousands place. Circle it.

20,000

The number to the right is an 8. Because 8 is greater than 5, increase the 9 by one and replace the digits that follow with 0's. ***Recall, when a 9 is increased it becomes a 10. This will require the number to the left of the 9 to be increased by one and the 9 will change to a 0.***

Rounding decimal numbers is quite similar to rounding integers. There is really only one difference. Any digits after the place value being considered will be dropped, if they occur <u>after</u> the decimal point. For example: round the number 127.6375 to the nearest hundredths.

127.6475

127.6④75

4 is the digit in the hundredths place. Circle it.

127.65

The number to the right is a 7. Because 7 is greater than 5, increase the 4 by one and drop all the digits after the decimal point.

It is important to note that when you round decimals, the final answer must have exactly the number of digits named.

Example 3

Round the following to the nearest <u>tenths</u>.

a. 16,047.346

16,047.③46

3 is the digit in the tenths place. Circle it.

16,047.3

The number to the right is a 4. Because 4 is less than 5, leave the 3 and drop all the digits after the tenths place.

b. 547.96

547.⑨6

9 is the digit in the tenths place. Circle it.

548.0

The number to the right is a 6. Because 6 is greater than 5, increase the 9 by one and drop all the digits after the tenths place. ***Remember the rule of rounding with 9, when the 9 is increased it becomes a 10. This will require the number to the left of the 9 to be increased by one and the 9 will change to a 0. Keep the last 0 so that the answer ends in the tenths place.***

Example 4

Round the following to the nearest <u>hundredths</u>.

a. 3,426.795

3,426.7⑨5

9 is the digit in the hundredths place. Circle it.

3,426.80

The number to the right is a 5. Because 5 is equal to 5, increase the 9 by one and drop all the digits after the hundredths place. ***Recall, when a 9 is the number in question, and it needs to be increased, the change also involves the digit to the left of the 9. Keep the last 0 so that the answer ends in tenths.***

b. 10.3428

10.3④28

4 is the digit in the tenths place. Circle it.

10.34

The number to the right is a 2. Because 2 is less than 5, leave the 4 and drop all the digits after the hundredths place.

Example 5

Round the following to the nearest <u>ones</u>.

a. 0.12

⓪.12

0 is the digit in the one's place. Circle it.

0

The number to the right is a 1. Because 1 is less than 5, leave the 0 and drop all the digits after the decimal point.

b. 15.63

1⑤.63

5 is the digit in the one's place. Circle it.

16

The number to the right is a 6. Because 6 is greater than 5, increase the 5 by one and drop all the digits after the decimal point.

Example 6

Round the following to the nearest cent.

a. $83.925

$\$83.9\textcircled{2}5$

$83.93

2 is the digit in the cent's place. (Cents is two decimal places.) Circle it.

The number to the right is a 5. Because 5 is equal to 5, increase the 2 by one and drop all the digits after the cent's place.

b. $47.884

$\$47.8\textcircled{8}4$

$47.88

8 is the digit in the cent's place. (Cents is two decimal places.) Circle it.

The number to the right is a 4. Because 4 is less than 5, leave the 8 and drop all the digits after the cent's place

Example 7

Complete the following table by rounding to the indicated place value.

Number	Thousands	Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths
855.4932	1,000	900	860	855	855.5	855.50	855.493
126.79	0	100	130	127	126.8	126.79	126.790
9.485732	0	0	10	9	9.5	9.49	9.486
0.127	0	0	0	0	0.1	0.13	0.127
0.0024	0	0	0	0	0.0	0.00	0.002
2,015	2,000	2,000	2,020	2,015	2,015.0	2,015.00	2,015.000
4,548.065	5,000	4,500	4,550	4,548	4,548.1	4,548.07	4,548.065

Now, let's look at where rounding can apply in our everyday activities. For instance, when dealing with money, we often find ourselves rounding to the nearest dollar. The following examples illustrate this idea.

Example 8

At the school bookstore you need to purchase a calculator for your math course. It costs $12.75. How many dollar bills do you need in order to make the purchase?

To determine the number of dollar bills needed, we must round $12.75 to the nearest dollar.

$12.75	
$1②.75	*2 is the digit in the dollar's place. Circle it.*
$13	*The number to the right is a 7. Because 7 is greater than 5, increase the 2 by one and drop all the digits after the dollar's place.*

Therefore, you need ***13 dollar bills*** *to make the purchase.*

The rules we have discussed are general rules for rounding. Now, closely examine the following example.

Example 9

It is Trevor's birthday and his roommates are throwing him a surprise party. Unfortunately, they forgot to purchase paper plates for the cake. Since they hate doing dishes, they decide to run to the local store to buy some paper plates. Trevor is very popular and his roommates expect about fifty-three people at the party. When they arrive to the store they find that the paper plates are sold in packages of ten. How many packages will they need to buy?

Since paper plates are sold in packages of 10, we must round 53 to the nearest tens. In our rounding rules, 53 rounds to 50.

The paper plates are sold in packages of 10. $50 \div 10 = 5$. *5 packages will be needed.*

Are 5 packages, 50 plates, enough? No. We will be short 3 plates. Therefore, Trevor's roommates must purchase an additional package giving them a total of ***6 packages, 60 plates.***

Example 8 is one application problem where the general rules of rounding do not necessarily apply. Carefully read your application problems to determine how to round.

1.3 EXERCISES

In 1-26 round to the indicated place value.

1. 13 *nearest tens*
2. 3 *nearest tens*
3. 48 *nearest tens*
4. 112 *nearest hundreds*
5. 987 *nearest hundreds*
6. 53 *nearest hundreds*
7. 2.47 *nearest tenths*
8. 123.2354 *nearest tenths*
9. 12.063 *nearest tenths*
10. 0.2745 *nearest hundredths*
11. 45.09864 *nearest hundredths*
12. 347.0005 *nearest thousandths*
13. 1,434.0572 *nearest thousandths*
14. 0.2476543 *nearest thousandths*
15. 0.67 *nearest ones*
16. 23.27 *nearest ones*
17. 243.504 *nearest ones*
18. $45.687 *nearest cent*
19. $2.0341 *nearest cent*
20. $4,305.706 *nearest cent*
21. $654.05 *nearest dollar*
22. $7,033.75 *nearest dollar*
23. $400.50 *nearest dollar*
24. $789.25 *nearest hundred dollar*
25. $12,532.00 *nearest hundred dollar*
26. $13,025.12 *nearest hundred dollar*

In 27-34, complete the following table by rounding to the indicated place value.

	Number	Thousands	Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths
27.	975.6342							
28.	12.345							
29.	5.68973							
30.	23,482.13							
31.	1005							
32.	98.0959							
33.	0.0348							
34.	5, 648.053							

In 35-39, solve the following application problems.

35. Beth and Tamee are planning a shopping trip to a large city over the Thanksgiving break, a distance of 543 miles. Estimate the distance to the nearest hundred miles.

36. Amie is doing her student teaching this semester. She wants to take treats to her school for twenty-three students. She goes to the grocery store and decides to buy candy bars. They are sold in packages of five. How many packages will she need to buy to have enough candy bars for her class? How many candy bars will be left over?

37. Cody goes to the local gravel pit. He knows that he needs to cover 439 square yards of ground with the gravel. One dump truck load will cover about 50 square yards. How many loads of gravel will he need?

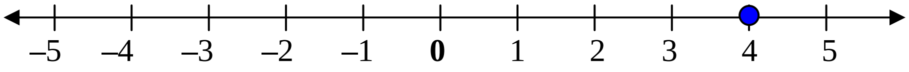
38. Danny needs to get his girlfriend a Valentines gift. He picked out a card that cost \$2.97, a box of chocolates that cost \$4.98, and a stuffed animal that costs \$6.99. How many five dollar-bills will he need to cover the cost of his Valentines gift?

39. Late in 1995, the national debt was reported to be approximately \$4,988,882,588,134. How would you write that number in words? How might you round that number so it would be easier to discuss and still convey the size of the number?

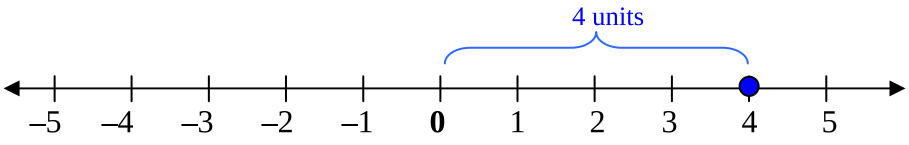
1.4 Absolute Value and Opposites

ABSOLUTE VALUE

The **absolute value** of a number is the distance that the given number is from zero on a number line. The notation for absolute value is $| \quad |$. For example, $|4|$ is read as "the absolute value of four." To find $|4|$, we will first draw a number line and plot the number 4.

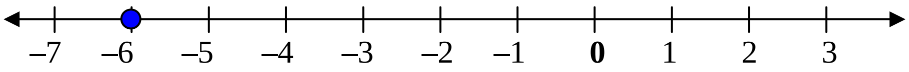


Now, find the distance from zero to 4.

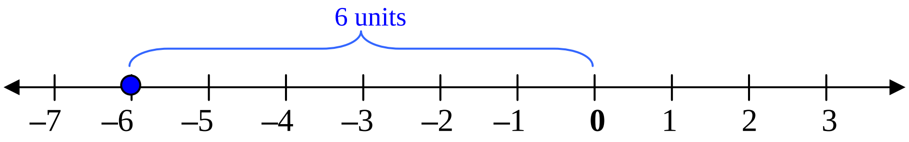


Therefore, $|4| = 4$

Now we will find the value of $|-6|$. We will first draw a number line and plot the number –6.



Now, find the distance from zero to –6.



Therefore, $|-6| = 6$.

Example 1

Find the value of $|5|$.

$|5| = 5$

5 is 5 units from zero on the number line, so it's absolute value equals 5.

Example 2

Find the value of $|-3|$.

$|-3| = 3$

–3 is 3 units from zero on the number line, so it's absolute value equals 3.

Example 3 **Find the value of $|-4.5|$.**

$|-4.5| = 4.5$

–4.5 is 4.5 units from zero on the number line, so it's absolute value equals 4.5.

Example 4 **Find the value of $|0|$.**

$|0| = 0$

0 is 0 units from zero on the number line, so it's absolute value equals 0.

Note: The absolute value of a number is the numbers distance from zero on a number line. Distance can never be a negative value. Therefore, the absolute value of a number is always equal to 0 or a positive number. An absolute value is never negative.

What is the difference between $|-6|$ and (-6)? We know that $|-6| = 6$ because the number –6 is 6 units from zero on the number line. When a number is enclosed in parentheses, ( ), it does not mean absolute value or distance, it simply means the number holds it's value. So, $(-6) = -6$

Example 5 **Find the value of (-5).**

$(-5) = -5$

Parentheses mean that the number holds it's value.

Example 6 **Find the value of (10).**

$(10) = 10$

Example 7 **Find the value of (-3.6).**

$(-3.6) = -3.6$

OPPOSITE

The opposite of a number is a number that is the same distance from zero on a number line but in the opposite direction. For example the opposite of 5 is –5.

The opposite of every positive number is a negative number and the opposite of every negative number is a positive number. Zero is its own opposite. To show the opposite we use a negative sign, –.

Example 8

Find the value of $-|15|$.

$-|15| = -15$

We are asked to find the opposite of the absolute value of 15. The absolute value of 15 is 15 and the opposite of 15 is –15.

Example 9

Find the value of $-(2)$.

$-(2) = -2$

We are asked to find the opposite of 2, which is –2.

Example 10

Find the value of $-\left|\frac{1}{4}\right|$.

$-\left|\frac{1}{4}\right| = -\frac{1}{4}$

We are asked to find the opposite of the absolute value of $\frac{1}{4}$. The absolute value of $\frac{1}{4}$ is $\frac{1}{4}$ and the opposite of $\frac{1}{4}$ is $-\frac{1}{4}$.

Example 11

Find the value of $-|-11|$.

$-|-11| = -11$

We are asked to find the opposite of the absolute value of –11. The absolute value of –11 is 11 and the opposite of 11 is –11.

Example 12

Find the value of $-(-7.5)$.

$-(-7.5) = 7.5$

We are asked to find the opposite of –7.5, which is 7.5.

Now that we know how to simplify absolute value problems and find the opposite of a number, we will compare these types of problems using the symbols: $>$, $<$, and $=$. Recall, the symbol $>$ means "is greater than," the symbol $<$ means "is less than," and the symbol = means "is equal to."

Example 13

Compare the following numbers. Insert $>$, $<$, or $=$ between the pair of numbers.

(-9) ____ $-|11|$

Simplify each number.

$-9 \underline{\;>\;} -11$

–9 is greater than –11.

Example 14

Compare the following numbers. Insert $>$, $<$, or $=$ between the pair of numbers.

$\lvert -4\rvert \underline{\qquad} \lvert 4\rvert$	*Simplify each number.*
$4 \underline{\quad = \quad} 4$	*4 is equal to 4.*

Example 15

Compare the following numbers. Insert $>$, $<$, or $=$ between the pair of numbers.

$-(21)\underline{\qquad}(-17)$	*Simplify each number.*
$-21 \underline{\quad < \quad} -17$	*–21 is less than –17.*

Example 16

Compare the following numbers. Insert $>$, $<$, or $=$ between the pair of numbers.

$-\lvert -5.4\rvert \underline{\qquad} -(-5.4)$	*Simplify each number.*
$-5.4 \underline{\quad < \quad} 5.4$	*–5.4 is less than 5.4.*

1.4 EXERCISES

In 1-16, simplify.

1. $|2|$
2. $|-10|$
3. (-5)
4. $-|-19|$
5. $|-0.3|$
6. $\left|\frac{1}{2}\right|$
7. $-(-7)$
8. $|0|$
9. $-|4|$
10. $|3.7|$
11. $-(8.3)$
12. $-|-7.5|$
13. $|8|$
14. $|1,247|$
15. (5.7)
16. $|-4,278|$

In 17-28, Compare the following numbers. Insert $>$, $<$, or $=$ ***between the pair of numbers.***

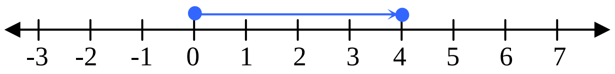
17. $-3 \underline{\qquad} -5$
18. $|-33| \underline{\qquad} -(-33)$
19. $-|17| \underline{\qquad} -(-17)$
20. $|0| \underline{\qquad} |-9|$
21. $-18 \underline{\qquad} -6$
22. $|-9| \underline{\qquad} |-14|$
23. $|-8| \underline{\qquad} |-4|$
24. $-45 \underline{\qquad} 0$
25. $-|-2| \underline{\qquad} -|-10|$
26. $-(-12) \underline{\qquad} -(-18)$
27. $-|-8| \underline{\qquad} -|-4|$
28. $-(-7) \underline{\qquad} -(-3)$

1.5 Adding & Subtracting Integers

ADDITION

When learning to add integers, start by drawing a number line. To the right of zero is always positive, to the left of zero is always negative. To add, you must always start at the origin, zero. Draw an arrow to represent your first value. If the value is positive, use an arrow pointing in the positive direction, right. If the value is negative, use an arrow pointing in the negative direction, left.

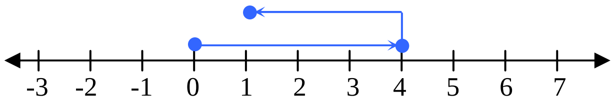
For example: $4+(-3)$



Start at zero and use an arrow pointing to the right with a length of 4.

Now, we will draw an arrow representing our second value. We will start at the tip of our first arrow, making sure to use an arrow pointing in the proper direction depending on the sign of the second value.

In the example, $4+(-3)$, we will start at the tip of the first arrow and draw a second arrow which will point in the negative direction, left. It will have a length of three units.

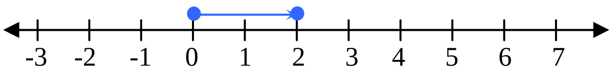


Draw the second arrow.

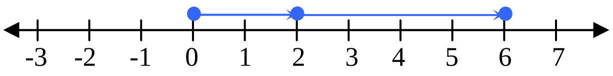
The point of the second arrow ends at the sum.

Therefore, $4+(-3)=1$

Example 1 **Simplify: 2 + 4**



Start at zero. Since the 2 is positive, draw an arrow 2 units long pointing in the positive direction, right.



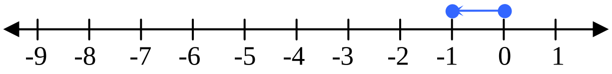
Beginning at the tip of the first arrow, draw a second arrow, 4 units long pointing in the positive direction, right.

2 + 4 = 6

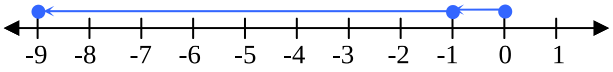
The point we end on is our answer, 6.

Example 2

Simplify: $(-1)+(-8)$



Start at zero. Since the 1 is negative, draw an arrow 1 unit long pointing in the negative direction, left.



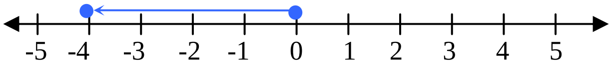
Beginning at the tip of the first arrow, draw a second arrow, 8 units long pointing in the negative direction, left.

$$(-1)+(-8)=-9$$

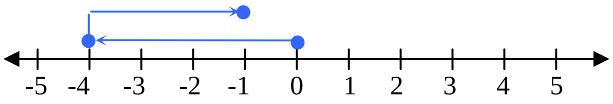
The point we end on is our answer, -9.

Example 3

Simplify: $-4+3$



Start at zero. Since the 4 is negative, draw an arrow 4 units long pointing in the negative direction, left.



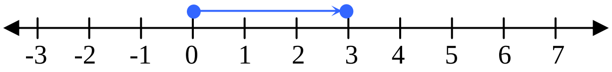
Beginning at the tip of the first arrow, draw a second arrow, 3 units long pointing in the positive direction, right.

$$-4+3=-1$$

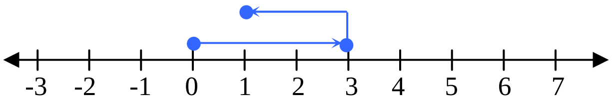
The point we end on is our answer, -1.

Example 4

Simplify: $3+(-2)$



Start at zero. Since the 3 is positive, draw an arrow 3 units long pointing in the positive direction, right.



Beginning at the tip of the first arrow, draw a second arrow, 2 units long pointing in the negative direction, left.

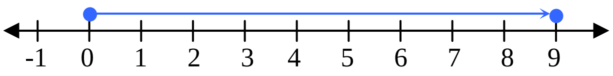
$$3+(-2)=1$$

The point we end on is our answer, 1

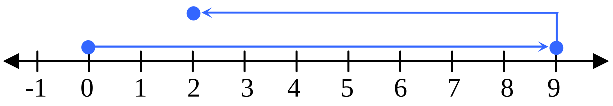
Did you notice that adding a negative, it is the same as subtracting a positive? The problem in example 4 can also be written as $3-2$. The answer will remain 1.

Example 5

Simplify: $9+(-7)$



Start at zero. Since the 9 is positive, draw an arrow 9 units long pointing in the positive direction, right.



Beginning at the tip of the first arrow, draw a second arrow, 7 units long pointing in the negative direction, left.

$$9+(-7)=2$$

The point we end on is our answer, 2

ADDITION OF INTEGERS

1. To add integers that have like signs, add the absolute value of the integers. The sum has the same sign as the original integers.
2. To add integers that have unlike signs, subtract the smaller absolute value from the larger absolute value. The sum has the sign of the integer with the greater absolute value.

The following is a list of words that represent the operation of addition:

ADDITION
add
sum
plus
total
increase
more
more *than**
combined
altogether
in all

the word **than will reverse the order of how the expression is written.*

Example 6

Find the sum of seven and negative eleven.

		The sum of	
Words:	*seven*	*and*	*negative eleven*
Expression:	7	$+$	(-11)

$$7+(-11)=-4$$

Example 7

Find five more than negative eight. *(The word* ***than*** *will reverse the order.)*

Words:	*five*	*more than*	*negative eight*
Expression:	-8	$+$	5

$$-8+5=-3$$

Example 8

Find the total of three, negative twelve and fourteen.

		total of			
Words:	*three*	*,*	*negative twelve*	*and*	*fourteen*
Expression:	3	$+$	(-12)	$+$	14

$$3+(-12)+14=5$$

SUBTRACTION

To subtract integers, rewrite your subtraction problem as an addition problem. To do this, take the first number and add the opposite of the second number. Examine the following examples.

subtraction problem	=	*first number*	+	*opposite of second number*		
$7-5$	$=$	7	$+$	(-5)	$=$	2
$-3-8$	$=$	-3	$+$	(-8)	$=$	-11
$4-(-1)$	$=$	4	$+$	1	$=$	5
$-12-(-6)$	$=$	-12	$+$	6	$=$	-6

SUBTRACTION OF INTEGERS

To subtract integers, rewrite your subtraction problem as an addition problem.

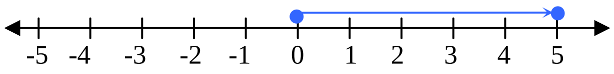
subtraction problem = *first number* + *opposite of second number*

Example 9

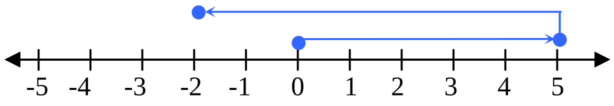
Simplify: $5-7$

Rewrite as an addition problem.

$5+(-7)$



Start at zero. Since the 5 is positive, draw an arrow 5 units long pointing in the positive direction, right.



Beginning at the tip of the first arrow, draw a second arrow, 7 units long pointing in the negative direction, left.

$5-7=-2$

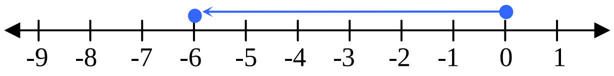
The point we end on is our answer, -2.

Example 10

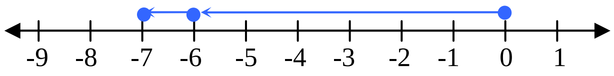
Simplify: $-6-1$

Rewrite as an addition problem

$-6+(-1)$



Start at zero. Since the 6 is negative, draw an arrow 6 unit long pointing in the negative direction, left.



Beginning at the tip of the first arrow, draw a second arrow, 1 unit long pointing in the negative direction, left.

$-6-1=-7$

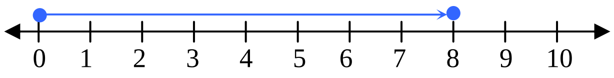
The point we end on is our answer, -7.

Example 11

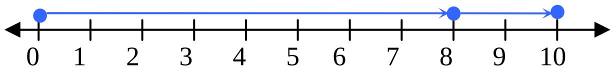
Simplify: $8-(-2)$

Rewrite as an addition problem.

$8+2$



Start at zero. Since the 8 is positive, draw an arrow 8 units long pointing in the positive direction, right.



Beginning at the tip of the first arrow, draw a second arrow, 2 units long pointing in the positive direction, right.

$8-(-2)=10$

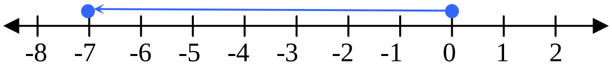
The point we end on is our answer, 10.

Example 12

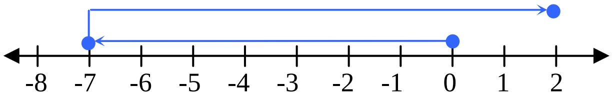
Simplify: $-7-(-9)$

Rewrite as an addition problem.

$-7+9$



Start at zero. Since the 7 is negative, draw an arrow 7 units long pointing in the negative direction, left.



Beginning at the tip of the first arrow, draw a second arrow, 9 units long pointing in the positive direction, right.

$-7-(-9)=2$

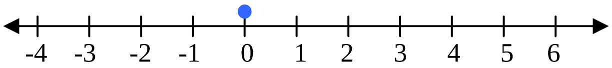
The point we end on is our answer, 2.

Example 13

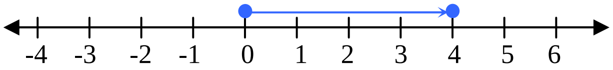
Simplify: $0-(-4)$

Rewrite as an addition problem.

$0+4$



Start at zero. Since the first number is zero, there is no need to draw an arrow.



Beginning at zero, draw an arrow, 4 units long pointing in the positive direction, right.

$0-(-4)=4$

The point we end on is our answer, 4.

The following is a list of words that represent the operation of subtraction:

SUBTRACTION
subtract
subtracted *from**
minus
difference
less
less *than**
decrease
reduce
remain
fewer
(other comparison words)

the word **than and **from** will reverse the order of how the expression is written.*

Example 14

Find the difference between ten and fifteen.

		The difference between	
Words:	*ten*	*and*	*fifteen*
Expression:	10	–	15

$$10-15=$$

$$10+(-15)=-5$$

Example 15

Find fourteen less five.

Words:	*fourteen*	*less*	*five*
Expression:	14	–	5

$$14-5=$$

$$14+(-5)=9$$

Example 16

Find fourteen less than five. *(The word* ***than*** *will reverse the order.)*

Words:	*fourteen*	*less than*	*five*
Expression:	5	–	14

$$5-14=$$

$$5+(-14)=-9$$

Example 17

Subtract eight from negative seven. *(The word* ***from*** *will reverse the order.)*

		Subtract	
Words:	*eight*	*from*	*negative seven*
Expression:	(-7)	–	8

$$(-7)-8=$$

$$(-7)+(-8)=-15$$

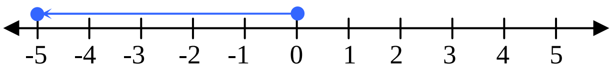
Now we will illustrate examples that involve addition and subtraction of more than two numbers.

Example 18

Simplify: $-5-0+10$

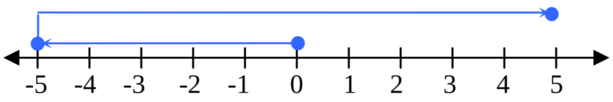
Rewrite as an addition problem.

$-5+0+10$



Start at zero. Since the 5 is negative, draw an arrow 5 units long pointing in the negative direction, left.

The second value is 0. There is no need to draw an arrow for this value.



Beginning at the tip of the first arrow, draw a second arrow, 10 units long pointing in the positive direction, right.

$-5 - 0 + 10 = 5$

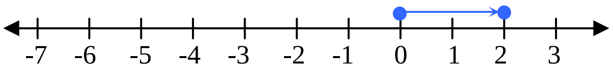
The point we end on is our answer, 5.

Example 19

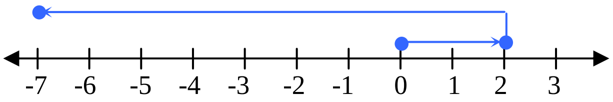
Simplify: $2-9+8-4$

Rewrite as an addition problem.

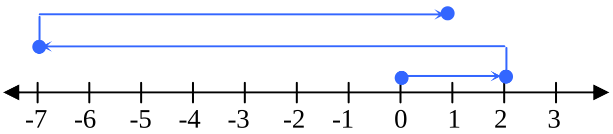
$2+(-9)+8+(-4)$



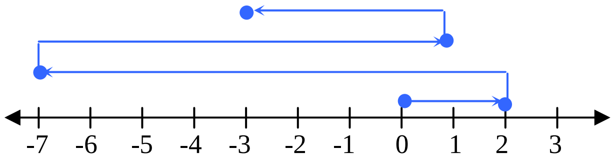
Start at zero. Since the 2 is positive, draw an arrow 2 units long pointing in the positive direction, right.



Beginning at the tip of the first arrow, draw a second arrow, 9 units long pointing in the negative direction, left



Beginning at the tip of the second arrow, draw a third arrow, 8 units long pointing in the positive direction, right.



Beginning at the tip of the third arrow, draw a fourth arrow, 4 units long pointing in the negative direction, left.

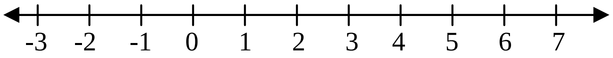
$2 - 9 + 8 - 4 = -3$

The point we end on is our answer, -3.

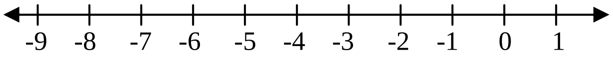
1.5 EXERCISES

In 1-6, use the given number line to simplify the following.

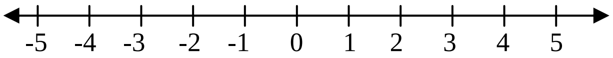
1. $3+4$



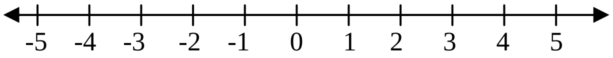
2. $(-2)+(-7)$



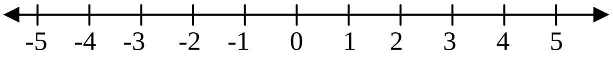
3. $5+(-8)+2$



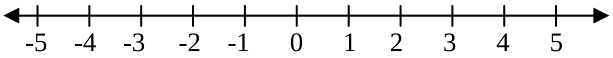
4. $2-6$



5. $0-5$



6. $4-1-(-2)$



In 7-22, simplify the following.

7. $8+(-3)$

8. $3+(-5)$

9. $(-8)+12$

10. $(-5)-8$

11. $4-(-6)$

12. $(-6)+(-2)$

13. $30-45$

14. $(-12)+3$

15. $(-7)-(-3)$

16. $(-6)-(-6)$

17. $5-2$

18. $(-1)+5+(-8)$

19. $(-6)-(-8)+(-12)-7$

20. $(-10)+(-5)-12$

21. $(-3)+4-(-23)-10$

22. $-10+14+25-16$

In 23-32, simplify the following.

23. Find the sum of five and negative ten.

24. Find two increased by seven.

25. Find negative eight more than three.

26. Combine four and negative five using addition.

27. Find the total of negative two, three, and negative one.

28. Find the difference between negative twenty and negative three.

29. Subtract eighteen from negative twenty-one.

30. Find negative eleven subtract two.

31. Find twelve less than negative ten.

32. Find twelve less negative ten.

In 33-37, solve the following application problems.

33. Mike has $125.00 in his checking account. He writes a check for $115.00, makes a deposit of $43.00, and writes another check for $57.00. Find the balance of his checking account.

34. The temperature on a February day is negative six degrees Celsius in the morning. If the temperature drops three degrees by 7:00 a.m., raises four degrees before 8:00 a.m., and drops seven degrees before 9:00 a.m. Find the temperature at 9:00 a.m.

35. Sharlie has $250.00 to spend on her books and school supplies this semester. Her Math book costs $63.00, her English book costs $52.00, her Biology book costs $85.00, and her calculator costs $22.00. She also purchases pencils, pens, paper and folders that cost a total of $13.00. How much money will Sharlie have remaining?

36. The average temperature on the surface of the Earth is fifteen degrees Celsius. The average temperature on the surface of Mars is negative sixty-three degrees Celsius. How many degrees warmer is the surface of the Earth than the surface of Mars?

37. Hailey went into the local office supply store. There she purchased a cellular telephone charger for $30.00 with a $13.00 mail-in rebate. She also purchased a box of compact discs for $18.00 with a $5.00 mail-in rebate. Finally, she purchased a box of folders for $10.00 with a $3.00 mail-in rebate. What was the total cost of Hailey’s purchases after the mail-in rebates?

1.6 Multiplying & Dividing Integers

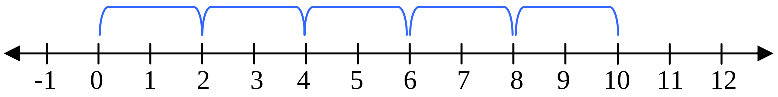
MULTIPLICATION

The operation of multiplication is a shorthand way of writing repeated addition. We know that

$$2\times 5=10$$

This actually means:

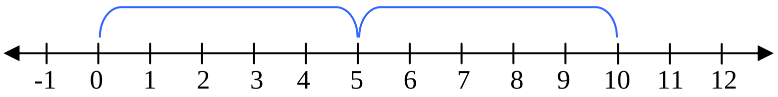
$$\underbrace{2+2+2+2+2}_{\textit{5 times}}=10$$



Notice:

$$5\times 2=10$$

$$\underbrace{5+5}_{\textit{2 times}}=10$$



2×5 is the same as 5×2. This concept is known as the Commutative Property of Multiplication.

COMMUTATIVE PROPERTY OF MULTIPLICATION

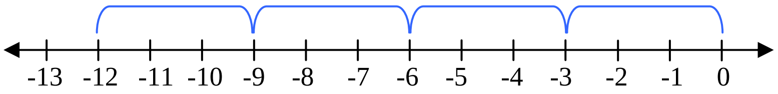
For all real numbers *a* and *b*,

$$a\cdot b=b\cdot a$$

When you multiply two values, the order in which you multiply does not matter.

Example 1 **Simplify:** -3×4

$$(-3)+(-3)+(-3)+(-3)=-12$$



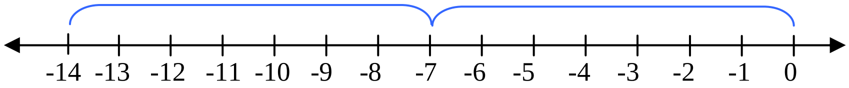
Example 2

Simplify: $2\times(-7)$

$(-7)\times 2$

$(-7)+(-7)=-14$

You cannot count negative groups. Use the Commutative Property of Multiplication to change the order.



In examples 1 and 2, we found that the product of a positive number and a negative number produces a negative number. We will now look at what happens with a negative number multiplied by a negative number.

$-2\times 3=-6$ *Notice, the values increase by 2 each time.*

$-2\times 2=-4$

$-2\times 1=-2$

$-2\times 0=0$ *Our product will increase by 2 giving us…*

$-2\times(-1)=2$

$-2\times(-2)=4$

$-2\times(-3)=6$

In the above example, we found the product of two negatives produces a positive answer. The saying “A negative times a negative equals a positive” will help you remember this concept.

MULTIPLICATION & DIVISION OF SIGNED NUMBERS

like signs
- positive ×/÷ positive = positive
- negative ×/÷ negative = positive

unlike signs
- positive ×/÷ negative = negative
- negative ×/÷ posititve = negative

Example 3

Find the product.

a) $6\times(-4)=-24$

b) $9\times 7=63$

c) $-12\times 3=-36$

d) $-10\times 0=0$

e) $(-5)\times(-7)=35$

f) $6\times(-3)=-18$

When finding the product of more than two numbers, the order in which you multiply does not matter. (The Associative Property of Multiplication). The following examples illustrate this concept.

Example 4

Find the product of $4(-5)(-2)$**.**

$$\underbrace{4(-5)}(-2)$$
$$(-20)(-2)$$
$$40$$

$$4\underbrace{(-5)(-2)}$$
$$4(10)$$
$$40$$

Example 5

Find the product.

a)
$$(-3)\underbrace{(-1)}(-11)$$
$$(3)(-11)$$
$$-33$$

b)
$$\underbrace{(-2)(3)}(5)$$
$$(-6)(5)$$
$$-30$$

c)
$$\underbrace{(-1)(-4)}\underbrace{(-3)(-2)}(4)$$
$$\underbrace{(4)\,(6)}\,(4)$$
$$(24)(4)$$
$$96$$

Note: If you have an odd number of negatives, your product will be negative.
If you have an even number of negatives, your product will be positive.

The following is a list of words that represent the operation of multiplication:

MULTIPLICATION

Multiply
Product
Times
Part Of
Twice
Area
Volume

Example 6

Find the product of two and eight.

		The product of	
Words:	*two*	*and*	*eight*
Expression:	2	×	8

$$2 \times 8 = 16$$

Example 7

Twice negative three multiplied by seven.

Words:	*twice*	*negative three*	*multiplied by*	*seven*
Expression:	2	(-3)	×	7

$$\underbrace{2(-3)}\times(7)$$

$$(-6)\times(7)$$

$$-42$$

Example 8

One-half of negative eight times negative seven.

Words:	*one – half*	*of*	*negative eight*	*times*	*negative seven*
Expression:	$\frac{1}{2}$	$\cdot$	(-8)	$\cdot$	(-7)

$$\underbrace{\frac{1}{2}\cdot(-8)}\cdot(-7)$$

$$(-4)\cdot(-7)$$

Example 9 **Multiply five, negative two, four, and negative one.**

$$\overbrace{\textit{five} \quad , \quad \textit{negative two} \quad , \quad \textit{four} \quad \textit{and} \quad \textit{negative one}}^{\textit{multiply}}$$

Words:	*five*	,	*negative two*	,	*four*	*and*	*negative one*
Expression:	5	$\cdot$	(-2)	$\cdot$	4	$\cdot$	(-1)

$$\underbrace{(5)(-2)}\underbrace{(4)(-1)}$$

$$(-10)(-4)$$

$$40$$

DIVISION

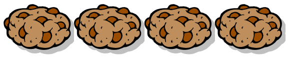
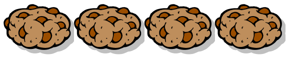
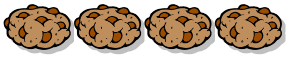
Suppose you have twelve cookies to divide evenly among four people. How many cookies will each person get?




There are four groups of three, so each person will get three cookies.

$$12 \div 4 = 3$$

Now, suppose that one person did not show up. How many cookies will each person get if we divide the twelve cookies evenly among three people?

There are three groups of four, so each person will get four cookies.

$$12 \div 3 = 4$$

The process of separating quantities into equal parts is called **division**. Division can be shown using several different types of notations. For example:

$$\begin{array}{r} 3 \\ 4\overline{)12} \end{array} \qquad 12 \div 4 = 3 \qquad \frac{12}{4} = 3$$

Division is the opposite or inverse operation of multiplication. In fact, division can be defined in terms of multiplication. The definition for division in general terms: For all real number a, b, and c where $b \neq 0$, $a \div b = c$ if and only if $c \cdot b = a$.

For example: $12 \div 4 = 3$ if and only if $3 \cdot 4 = 12$.

MULTIPLICATION & DIVISION OF SIGNED NUMBERS

like signs
- positive ×/÷ positive = positive
- negative ×/÷ negative = positive

unlike signs
- positive ×/÷ negative = negative
- negative ×/÷ posititve = negative

Example 10

Simplify: $100 \div (-25)$

$100 \div (-25) = -4$

Example 11

Simplify: $-132 \div (-11)$

$-132 \div (-11) = 12$

Example 12

Simplify: $-215 \div 5$

$-215 \div 5 = -43$

Example 13

Simplify: $120 \div (-2) \div (-3)$

$120 \div (-2) \div (-3)$ *Work left to right.*

$-60 \div (-3)$

20

When working with division, we need to recall the property of division by zero introduced in 1.1.

Division by Zero Property

For all non-zero real numbers *a*,

$$a \div 0 = \frac{a}{0} = \text{undefined}$$

Why is division of non-zero numbers undefined? Let's look at the example $12 \div 0$. What do you think the answer should be? Did you think it was 12 or maybe 0? Using the definition of division would either of these two answers be correct? Let's start with 12.

$12 \div 0 = 12 \quad \textit{if and only if} \quad 12 \cdot 0 = 12$ We know that $12 \cdot 0 = 0$ and $0 \neq 12$, so this answer could not be correct.

What about 0?

$12 \div 0 = 0 \quad \textit{if and only if} \quad 0 \cdot 0 = 12$ We know that $0 \cdot 0 = 0$ and $0 \neq 12$, so this answer could not be correct.

If you continue to try values, you will never find a number that works. Therefore, d**ivision of nonzero numbers by 0 is always undefined.**

Example 14

Simplify: $35 \div 0$

$35 \div 0 = \textit{undefined}$

Example 15

Simplify: $0 \div 35$

$0 \div 35 = 0$ *Division into zero equals zero.*

The following is a list of words that represent the operation of division:

DIVISION

Divide
Quotient
Divided By
Divided *Into**
Split
Each
Shared
Per
Equal Parts
Ratio
Average

the word **into will reverse the order of how the expression is written.*

Example 16

Find the quotient of ninety-nine and eleven.

		The quotient of	
Words:	*ninety – nine*	*and*	*eleven*
Expression:	99	÷	11

$$99 \div 11 = 9$$

Example 17

Find the ratio of fifty-six and negative eight.

		The ratio of	
Words:	*fifty – six*	*and*	*negative eight*
Expression:	56	÷	(-8)

$$56 \div (-8) = -7$$

Example 18

		Divide	
Words:	*negative ninety – six*	*by*	*twelve*
Expression:	(-96)	÷	12

$$(-96) \div 12 = -8$$

Example 19

Divide negative six into negative seventy-two.

(The word ***into*** *will reverse the order.)*

		Divide	
Words:	*negative six*	*into*	*negative seventy – two*
Expression:	−72	÷	−6

$$(-72) \div (-6) = 12$$

1.6 EXERCISES

In 1-20, simplify the following.

1. $\left(-4\right)9$

2. $\frac{12}{4}$

3. $0 \div 5$

4. $\frac{\left(-36\right)}{3}$

5. $\left(-7\right)\left(-2\right)$

6. $2 \div 0$

7. $\frac{\left(-49\right)}{\left(-7\right)}$

8. $\left(-9\right) \cdot 6$

9. $0\left(14\right)$

10. $480 \div \left(-8\right)$

11. $\frac{56}{8}$

12. $10 \cdot \left(-5\right) \cdot 0$

13. $240 \div \left(-40\right)$

14. $\left(-2\right) \times 3 \times \left(-7\right)$

15. $6\left(-5\right)\left(-2\right)$

16. $100 \div 5 \div 2$

17. $3 \times \left(-2\right) \times \left(-7\right)$

18. $\left(-20\right)\left(5\right)\left(2\right)$

19. $\left(-1\right)\left(2\right)\left(7\right)\left(-3\right)$

20. $225 \div 25 \div 3$

In 21-28, simplify the following.

21. Find the product of two and four.

22. Multiply two, four, negative three, and one.

23. Twice negative two multiplied by eight.

24. Three times the product of negative two and one.

25. Find the quotient of sixty-six and three.

26. Divide two hundred twenty-five by negative fifteen.

27. Divide negative three into twenty-seven.

28. Find the ratio of eighteen and three.

In 29-31, solve the following application problems.

29. A weather forecaster predicts that the temperature will drop five degrees each hour for the next six hours. Represent this drop as a product of integers and find the total drop in temperature.

30. Joe lost $400.00 on each of seven consecutive days in the stock market. Represent his total loss as a product of integers and find his total loss.

31. A card player had a score of negative twelve for the total of four rounds in a game. Find her average score for each round.

1.7 REVIEW EXERCISES

1. What is the difference between a whole number and a natural number?

2. What is the difference between real numbers and integers?

3. If you multiply by zero, what is the product?

4. If you divide by zero, what is the quotient?

In 5-6, write the following numbers in words.

5. 17.2 ____

6. –3,247,000.386 ____________________________________

In 7-8, change from words into digits.

7. One hundred forty-three and nine hundredths ______________________

8. Two thousand, eleven and four hundred fifty-eight hundred-thousandths__________

In 9-12, Compare the following numbers. Insert $>$, $<$, or $=$ ***between the pair of numbers.***

9. 3.12____3.1

10. $-(-7)$ ____ $-|-7|$

11. $-|-2|$ ____ $-|2|$

12. $|3|$ ____ (-3)

In 13-16, complete the following table by rounding to the indicated place value.

	Number	**Thousands**	**Hundreds**	**Tens**	**Ones**	**Tenths**	**Hundredths**	**Thousandths**
13.	13.543							
14.	762.48							
15.	-14.095							
16.	3,457.2834							

In 17-42, simplify the following.

17. $|-7.5|$

18. $\left(-\frac{3}{4}\right)$

19. $-|1|$

20. $-(-6)$

21. $-\left|-\frac{1}{3}\right|$

22. $(-2)+7$

23. $15-17$

24. $1-3-(-4)$

25. $(-4)-(-3)+9-0$

26. Find fourteen less than seven.

27. Find eight increased by three.

28. Subtract one and negative four.

29. Find negative two more than three.

30. $(-7)(-8)$

31. $\frac{(-108)}{12}$

32. $(-3)(-6)(-1)(10)$

33. $111 \div 3$

34. $4 \times 2 \times (-1)$

35. $13 \div 0$

36. $\frac{0}{(-4)}$

37. $150 \div (-3) \div (2)$

38. Divide five thousand, ninety-two by four.

39. Find the product of negative twelve and eleven.

40. Find the quotient of negative seventy-two and negative twelve.

41. Multiply three by fourteen.

42. Divide zero into five.

In 43-44, solve the following application problems.

43. Tanner has \$168.00 in his checking account. On Monday, he makes a deposit of \$25.00. On Tuesday, he writes a check for \$37.00. On Wednesday, he writes a check for \$74.00. On Friday he writes a check for \$6.00. What is the balance in Tanner’s checking account at the end of the week?

44. Natalie spent \$4.00 for her lunch five days in a row. What is the total amount Natalie spent for her lunches during the five-day period?