5.1 Integer Exponents & Scientific Notation

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Objectives:

- Use the product rule for exponents.
- Define 0 and negative exponents.
- Use the quotient rule for exponents.
- Use the power rules for exponents.
- Simplify exponential expressions.
- Use the rules for exponents with scientific notation.

Product Rule for Exponents

- Consider:

$$2^5 \cdot 2^3 =$$

$$x^4 \cdot x^7 =$$

Product Rule for Exponents

If m and n are natural numbers and a is any real number, then

When multiplying like bases, keep the base and add the exponents.

Example 1

- Apply the produce rule, if possible, in each case.

a) $m^8 \cdot m^6$

b) $(-5y^4)(-9y^5)$

c) $m^5 \cdot p^4$

d) $(-3x^2y^3)(7xy^4)$

Zero Exponent

- Consider:

$$4^2 \cdot 4^0 =$$

Zero Exponent

If a is any nonzero real number, then

Any non-zero base raised to the power of 0 equals 1.

The expression 0^0 is undefined.

Example 2

- Evaluate.

a) 5^0

b) -5^0

c) $(-5x)^0$

d) $10^0 - 9^0$

Negative Exponents

- Consider:

$$8^2 \cdot 8^{-2} =$$

Negative Exponents

For any natural number n and any nonzero real number a,

Negative exponents lead to reciprocals.

- Write with only positive exponents:

a) 6^{-5}

b) $-7p^{-4}$

Example 3

- Write with only positive exponents.

a) $(5z)^{-3}$

b) $5z^{-3}$

c) $(2x)^{-4}$

d) $4^{-1} - 2^{-1}$

Special Rules for Negative Exponents

For any natural number n and any nonzero real number a,

Negative exponents lead to reciprocals.

- Evaluate.

a) $\dfrac{1}{4^{-3}}$

b) $\dfrac{3^{-3}}{9^{-1}}$

Quotient Rule for Exponents

- Consider:

$$\frac{a^8}{a^3}$$

Quotient Rule for Exponents

If a is any nonzero real number and m and n are integers, then

When dividing like bases – keep the base and subtract the exponents.
(top exponent – bottom exponent)

Example 4

- Write with only positive exponents.

a) $\frac{x^7}{x^2}$

b) $\frac{5^{-6}}{5^{-8}}$

c) $\frac{m^8}{m^{13}}$

d) $\frac{x^3}{y^5}$

Power Rule for Exponents

- Consider:

$(3^4)^2 =$

$(xy)^4 =$

$\left(\frac{2}{y}\right)^4 =$

Power Rules for Exponents

If a and b are real numbers and m and n are integers, then

To raise a power to a power, multiply the exponents.

To raise a product to a power, raise each factor to that power.

To raise a quotient to a power, raise the numerator and denominator to that power.

Example 5

- Simplify using the power rules.

a) $(r^5)^4$

b) $\left(\frac{3}{4}\right)^3$

c) $(-3y^5)^2$

d) $\left(-\frac{2m^5}{z}\right)^3$

Special Rules for Negative Exponents (Continued)

If $a \neq 0$ and $b \neq 0$ and n is an integer, then

Any non-zero number raised to the negative nth power is equal to the reciprocal of that number raised to the nth power.

- Write with only positive exponents then evaluate.

a) $\left(\frac{2}{3}\right)^{-4}$

b) $\left(\frac{4x}{5}\right)^{-3}$

Example 6

- Simplify each exponential expression so that no negative exponents appear in the final answer.

a) $x^{-4} \cdot x^{-6} \cdot x^{8}$

b) $\dfrac{(2m^2n)^2}{m^3n^2}$

Example 7

- Simplify each exponential expression so that no negative exponents appear in the final answer.

a) $\left(\frac{3x^2}{y}\right)^2 \left(\frac{4x^3}{y^{-2}}\right)^{-1}$

b) $\left(-\frac{4m^5n^4}{24mn^{-7}}\right)^{-2}$

Scientific Notation

- Numbers occurring in science are often extremely large (such as the distance from Earth to the Sun, 93,000,000 miles) or extremely small (the wavelength of yellow-green light, approximately 0.0000006 m.) Because of the difficulty of working with many zeros, scientists often express such numbers with exponents, using a form called scientific notation.

Scientific Notation

A number is written in **scientific notation** when it is expressed in the form

Scientific Notation

- Using your calculator (or cell phone) find:

$$5{,}000{,}000 \times 6{,}000{,}000{,}000{,}000$$

- Numbers in scientific notation with _____ exponents are relatively _____.

- Numbers in scientific notation with _____ exponents are relatively _____.

Example 8

- Write each number in scientific notation.

a) 0.0571

b) $-2{,}140{,}000{,}000$

c) 843

d) 0.000062

Example 9

- Write each number in standard notation.

a) 2.51×10^{3}

b) -6.8×10^{-4}

c) -5.7×10^{-2}

d) 1.083×10^{0}

Example 10

- Evaluate. Write answers in scientific notation and in standard notation.

a) $(3 \times 10^{5})(5 \times 10^{-2})$

b) $\dfrac{4.8 \times 10^{2}}{2.4 \times 10^{5}}$

Example 11

- Evaluate. Write answers in scientific notation and in standard notation.

$$\frac{1{,}920{,}000 \times 0.0015}{0.000032 \times 45{,}000}$$

Example 12

- If the speed of light is approximately 3.0×10^5 km/sec, how many seconds does it take light to travel approximately 1.5×10^8 km from the Sun to the Earth?