Derivatives, Slopes, Tangent Lines, and Making Movies

Note: You may notice differences between this Maple worksheet and the equivalent Mathematica notebook. These differences were introduced to preserve the content of these modules and were necessary because of major functional differences between Maple and Mathematica.

Introduction

OBJECTIVE: To visualize the derivative and the linearization of a function at a point.

In this module, we explore the derivative as the slope of a nonlinear function and find the equation of the line tangent to a curve at a point. You will learn how to plot the curve and selected tangents on the same graph. In addition, you will see how to use *Maple* to make a movie animation by generating a sequence of plots, each showing a different tangent to the curve. When the sequence of graphs is animated, the tangent lines appear to roll along the graph of the function.

Technology Guidelines

NOTE: If you have just finished a worksheet, **restart** *Maple* before executing a new worksheet.
TO OPEN SECTIONS,
Click on the **PLUS** sign at the left hand side of the screen *or* select **Expand All Sections** from the **View** drop down menu.

TO STOP AN EXECUTION
Click on **STOP** button from the toolbar.

ORDER OF EXECUTION
Execute commands in the order given. Do not skip any *Maple* Input lines within a given worksheet

Alternatively, you can execute the entire worksheet by selecting the **Execute Worksheet** command from the **Edit** drop down menu.

SAVING WORKSHEETS.

You can save anytime to any directory you choose, and it is wise to save often.
EXPERIENCING MAJOR PROBLEMS
Save if appropriate, and then sut down *Maple* and start it up again.

You Try It

First, work through Parts I - V with the example function $f(x) = x^2$, and then repeat the steps in

Parts I - V for some functions that you select. Here are some suggestions.

1. x^3 for $-2 <= x <= 2$

2. $\sin(x)$ for $0 <= x <= 2\pi$

3. $e^{(-x^2)}$ for $-2 <= x <= 2$

4. $\sqrt{x}$ for $0 < x \leq 4$ Note that we do not include $x = 0$. Do you know why?

Part I: The Derivative at a Point

Chapter 3, Section 1

Define a nonlinear function of your choice and call it $f(x)$ and graph it. For an example, we choose the function $f(x) = x^2$ for $-2 <= x <= 2$ (To put in a different function and domain, change the entries in the following input cell.)

```
> restart;
  with(plots):
  x0:=-2;
  xf:=2;
  f:=x->x^2;
```

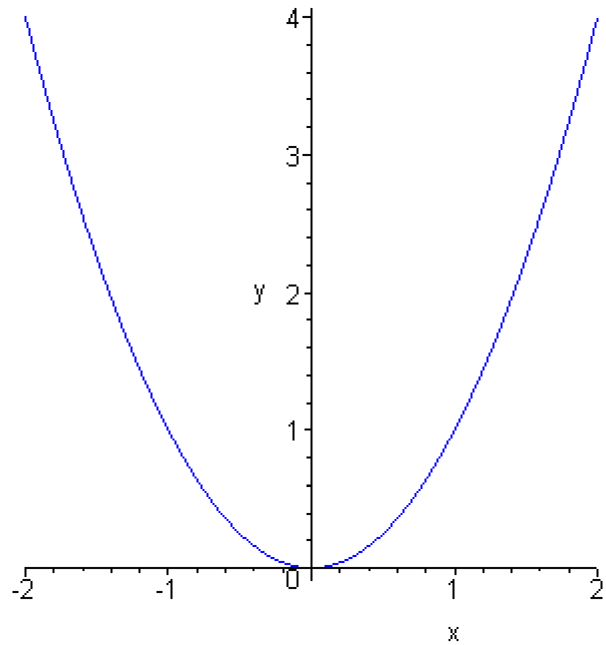
Warning, the name changecoords has been redefined

$$x0 := -2$$

$$xf := 2$$

$$f := x \rightarrow x^2$$

```
> plot(f(x), x=x0..xf, labels=["x","y"], color=blue);
```



Now pick a point on the function and use the definition of the derivative to find the slope of the function's graph at the point you pick, calling it $m_{tangent}$. We choose $x = 1$ for our example.

```
> mtangent:=limit((f(1+h)-f(1))/h,h=0);
```

$$mtangent := 2$$

Part II: The Linearization of a Function

Chapter 3, Sections 1 and 8 (ET Sections 1 and 10)

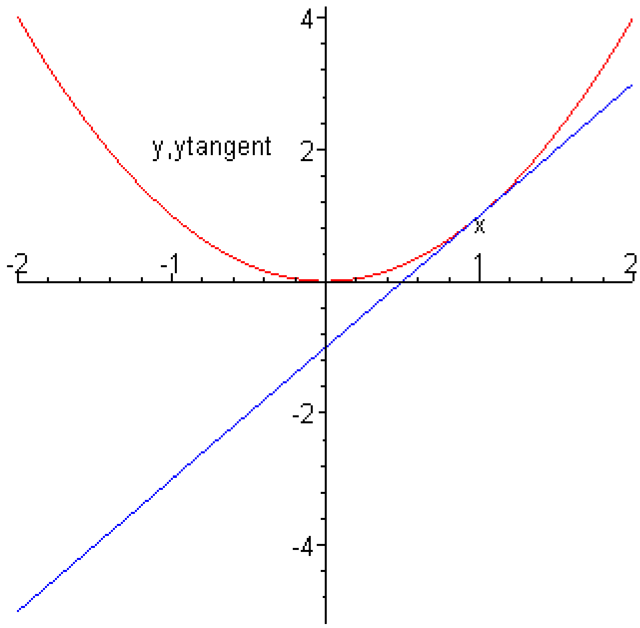
Form a new function for the line that is tangent to the function that you chose in the "You Try It" section at that point you picked in Part I, and call it $y_{tangent} = L(x)$. This function is called the *linearization* of $y = f(x)$ at the point $(1, f(1))$.

```
> L:=x->f(1)+mtangent*(x-1);
```

$$L := x \to f(1) + mtangent\,(x - 1)$$

Plot the function and the tangent line on the same graph.

```
> plot({f(x), L(x)}, x=x0..xf, labels=["x","y,ytangent"], color=[blue, red]);
```



Part III: The Derivative Function

Chapter 3, Section 1

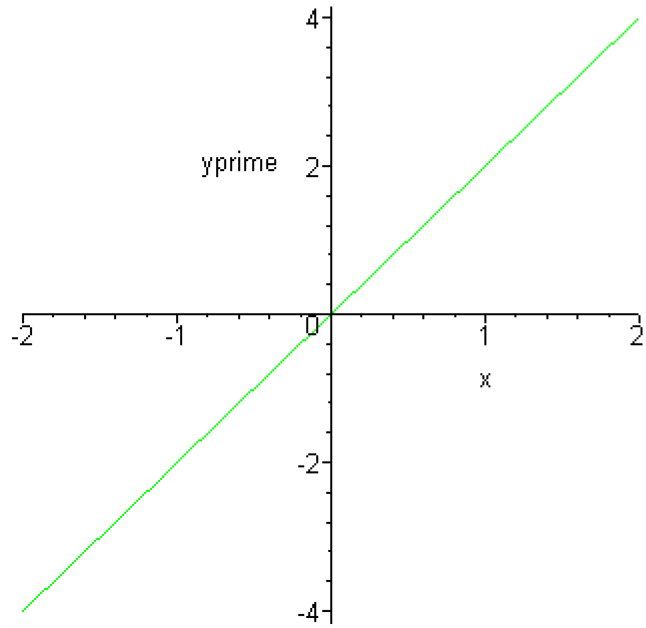
Form the derivative function that will give the slope of the tangent to your chosen function at any point with coordinates $(x, f(x))$. Call the derivative function **yprime(x)**, and graph it.

```
> yprime:=limit((f(x+h)-f(x))/h,h=0);
```

$$yprime := 2\,x$$

Graph ***yprime(x)*.**

```
> plot(yprime, x=x0..xf, labels=["x", "yprime"], color=COLOR(RGB,0,1,0));
```



Part IV: A Whole Bunch of Tangents

Chapter 3, Section 1

Form a new *Maple* function that gives the equation of the line tangent to $y = f(x)$ at the point $(a, f(a))$. Call the new function **tanline(x,a)**.

```
> tanline:=(x,a)->f(a)+eval(yprime,x=a)*(x-a);
```

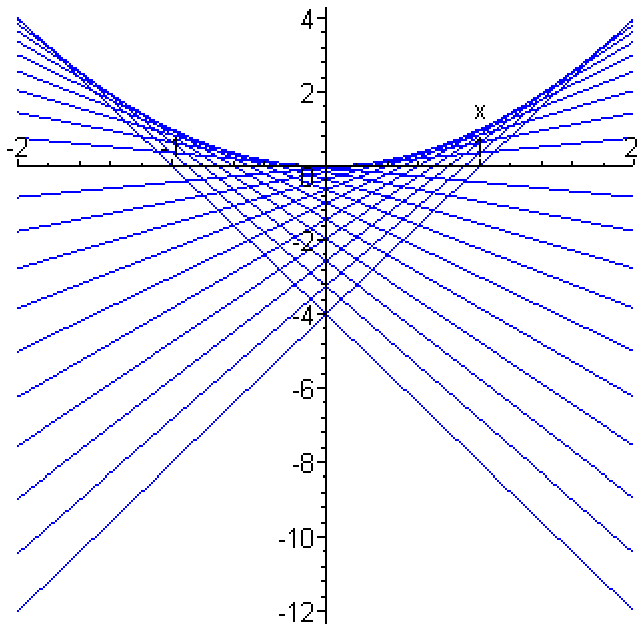
$$tanline := (x, a) \to f(a) + yprime\Big|_{x=a} (x - a)$$

Test your **tanline(x,a)** for several values of a by plotting the tangent lines and $y = f(x)$ together on the same graph. First, use the **tanline(x,a)** function and a short loop to generate a list of equations for the tangents to the curve at points $(a, f(a))$, for values of a varying from -2 to 2 in increments of 0.1. Then graph the tangent lines and $y = f(x)$ together on the same graph.

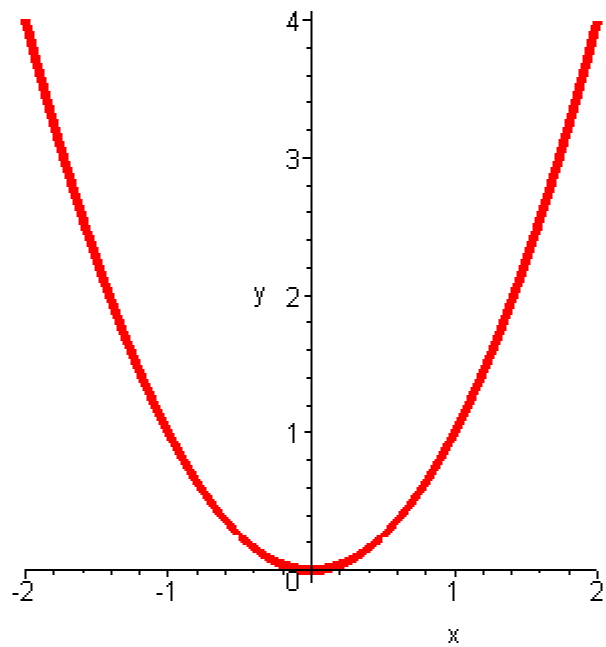
```
> listoftangentline:=seq(evalf(tanline(x,x0+(xf-x0)*a/20)), a=0..20);
```

$listoftangentline := -4. - 4.\ x, -3.240000000 - 3.600000000\ x, -2.560000000 - 3.200000000\ x,$
$-1.960000000 - 2.800000000\ x, -1.440000000 - 2.400000000\ x, -1. - 2.\ x, -0.6400000000 - 1.$
$-0.3600000000 - 1.200000000\ x, -0.1600000000 - 0.8000000000\ x, -0.04000000000 - 0.40000$
$-0.04000000000 + 0.4000000000\ x, -0.1600000000 + 0.8000000000\ x, -0.3600000000 + 1.2000$
$-0.6400000000 + 1.600000000\ x, -1. + 2.\ x, -1.440000000 + 2.400000000\ x, -1.960000000 + 2.$
$-2.560000000 + 3.200000000\ x, -3.240000000 + 3.600000000\ x, -4. + 4.\ x$

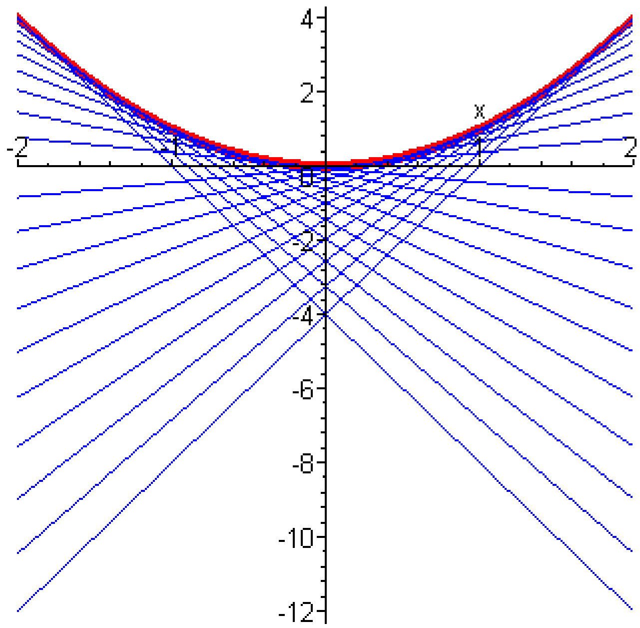
```
> p1:=plot({listoftangentline}, x=x0..xf, color=blue):
  print(p1);
```



> **p2:=plot(f(x), x=x0..xf, thickness=[5], labels=["x","y"]): print(p2);**



> **print(plots[display]({p1,p2}));**

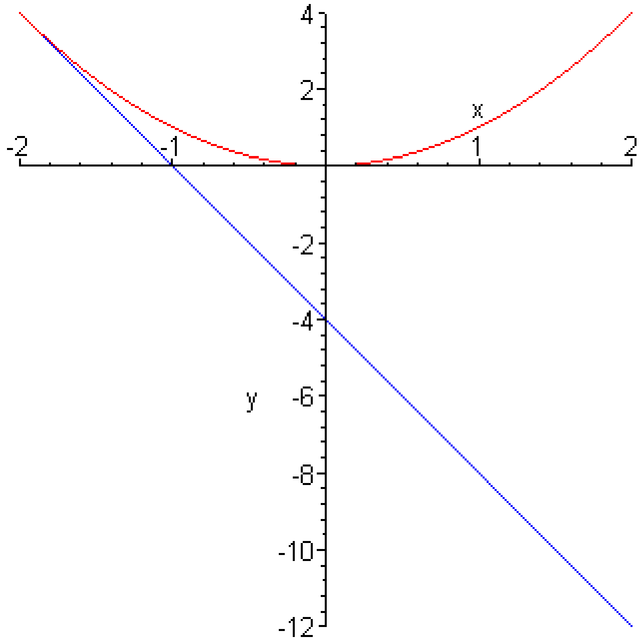


Part V: Making Movies

Chapter 3, Section 1

The following command generates a sequence of graphs that you can animate. Click on the graph below to bring up the animation toolbar at the top of the worksheet.

```
> tanlist:=[listoftangentline]:
  b:=1:
  tanplot := []:
  for a from evalf(x0) to evalf(xf) by evalf((xf-x0)/20) do
     tanplot := [op(tanplot), plot([f(x),tanlist[b]], x=x0..xf,color=[red,blue])]:
     b:=b+1:
  od:
  display(seq(plot([f(x),listoftangentline[i]], x=x0..xf,labels=["x","y"],color=
  [red,blue]),i=1..21),insequence=true);
```


4
2
x
-2
-1
1
2
-2
-4
y
-6
-8
-10
-12

>