PHSX 101L **Experiment 7** **Name:**_____

Title: Velocity Measurement Using a Ballistic Pendulum

Object: To determine the velocity of a projectile by two independent methods: (1) measuring a trajectory, and (2) using a ballistic pendulum; and then to compare the results.

Theory: When a projectile (of mass m) is fired into a ballistic pendulum (of mass M) the impact causes the pendulum and embedded projectile (total mass $M + m$) to swing up to a maximum height. Kinetic energy is not conserved in the inelastic collision, but momentum is. And after the collision mechanical energy is conserved as the kinetic energy of the pendulum immediately after the impact is changed completely into potential energy at the maximum height. These conservation laws can be used to derive an expression for the initial velocity v of the projectile.

Conservation of angular momentum during the collision gives:

$$\begin{aligned} \Sigma L_i &= \Sigma L_f \\ Rmv &= I\omega \\ v &= \frac{I\omega}{Rm} \end{aligned}$$

where I is the moment of inertia and R is the distance from a horizontal axis of rotation of the pendulum to the projectile.

Conservation of mechanical energy after the collision gives:

$$\begin{aligned} \Sigma E_i &= \Sigma E_f \\ \frac{1}{2}I\omega^2 &= (M+m)gh \\ \omega^2 &= \frac{2gh(M+m)}{I} \\ \omega &= \sqrt{\frac{2gh(M+m)}{I}} \end{aligned}$$

where $h = d(1-\cos\theta)$ is the vertical rise of the center of mass of the pendulum (including projectile), and d is the distance from the horizontal axis to the center of mass.

Combining these two results into one equation we get:

$$v = \sqrt{\frac{2gh(M+m)I}{R^2m^2}}. \tag{1}$$

The velocity of a projectile can also be determined from measurements made on its free-flight trajectory fired horizontally:

$$x = v_o t \qquad\qquad y = \frac{1}{2} g t^2$$

$$v_o = x/t \qquad\qquad t = \sqrt{\frac{2y}{g}}$$

$$v_o = \sqrt{\frac{gx^2}{2y}} \tag{2}$$

In this formula x is the horizontal range of the projectile, and y is the vertical distance through which it falls.

Apparatus: Make a drawing of the apparatus used.

Procedure and Results:

1. Determine mass of projectile m: _____
2. Determine mass of pendulum M: _____
3. Find center of mass of pendulum L: _____
4. Determine the rotational inertia of pendulum (including projectile) I: _____
5. Fire projectile into pendulum and measure maximum angle of swing θ: _____
6. Calculate rise of center of mass of pendulum $h = L(1 - cos\theta)$: _____
7. Use equation from pendulum theory to calculate projectile velocity v: _____
8. Fire projectile off table and measure horizontal range of trajectory x: _____
9. Also measure vertical distance of fall of trajectory y: _____
10. Use equation from trajectory theory to calculate projectile's velocity v: _____
11. Percent difference between the velocities determined by the two methods: _____

Questions:

1. Do the two methods of determining velocity agree within expected error limits? Explain.

2. Which method do you consider to be most reliable? Why?

Conclusions: