

Title: Velocity Measurement Using a Ballistic Pendulum

Object: To determine the velocity of a projectile by two independent methods: (1) using a ballistic pendulum, and (2) measuring a trajectory; and then to compare the results.

Theory: When a projectile (of mass m_b) is fired into a ballistic pendulum (of mass m_c) the impact causes the pendulum and embedded projectile (total mass $m_b + m_c$) to swing up to a maximum height h . Kinetic energy is not conserved in the inelastic collision, but momentum is. And after the collision mechanical energy is conserved as the kinetic energy of the pendulum immediately after the impact is changed completely into potential energy at the maximum height. These conservation laws can be used to derive an expression for the initial velocity v_o of the projectile.

Conservation of momentum *during* the collision gives:

$$m_b v_o = (m_b + m_c) v \quad (1)$$

where m_b is the mass of the ball, v_o is the muzzle velocity of the ball, m_c is the mass of the catcher, and v is the velocity of the catcher (and ball) after collision.

Conservation of mechanical energy *after* the collision gives:

$$\frac{1}{2}(m_b + m_c)v^2 = (m_b + m_c)gh. \quad (2)$$

Where g is the acceleration of objects in free-fall (which we called a_g in a previous lab) and h is height above the starting point. Solving (2) for v we get:

$$v = \sqrt{2gh}. \quad (3)$$

Substitute (3) into (1):

$$m_b v_o = (m_b + m_c) \sqrt{2gh} \quad (4)$$

$$v_o = \left(\frac{m_b + m_c}{m_b} \right) \sqrt{2gh}. \quad (5)$$

The velocity of a projectile can also be determined from measurements made on its free-flight trajectory fired horizontally. In these formulae x is the horizontal range of the projectile, and y is the vertical distance through which it falls, and t is the time of flight (which is eliminated in equation 6).

$$\begin{aligned} x &= v_o t & y &= \frac{1}{2} g t^2 \\ v_o &= x/t & t &= \sqrt{\frac{2y}{g}} \\ v_o &= \sqrt{\frac{g x^2}{2y}} & & (6) \end{aligned}$$

Apparatus: Make a drawing of the apparatus used.

Procedure and Results:

For the pendulum method of determining v_o :

1. Determine mass of ball m_b : _____
2. Determine mass of catcher m_c : _____
3. Fire the projectile into the catcher and measure the rise h : _____
4. Calculate v_o from equation 5: _____

For the projectile method of determining v_o :

5. Measure vertical distance of fall y : _____
6. Fire the projectile ten times and find the average horizontal range x : _____
7. Use equation 6 from trajectory theory to calculate v_o : _____
8. Percent difference between the velocities determined by the two methods: _____

Questions:

1. Do the two methods of determining velocity agree within expected error limits? Explain.
2. Which method do you consider to be most reliable? Why?

Conclusions: