Sec 6.4 cont.

$\sin\theta = \frac{y}{r}$ $\quad \cos\theta = \frac{x}{r}$ $\quad \tan\theta = \frac{y}{x}$

$y = r\sin\theta$ $\quad x = r\cos\theta$ $\quad x^2 + y^2 = r^2$

Given Rect (x, y)

$\tan\theta = \frac{y}{x}$ $\quad x^2 + y^2 = r^2$

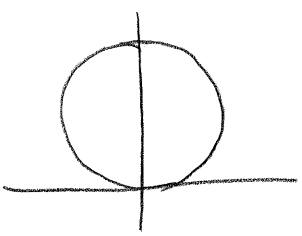
↑ find θ $\quad$ ↑ find r

Given Polar (r, θ)

$x = r\cos\theta$; $y = r\sin\theta$

↑ find x $\quad$ ↑ find y

$r = 4\sin\theta$ $\xrightarrow{\text{multiply by } r}$ $r^2 = 4r\sin\theta$



$\downarrow$

$x^2 + y^2 = 4y$

Rect St form of circle

$(x-h)^2 + (y-k)^2 = r^2$

$x^2 + y^2 - 4y + 4 = 0 + 4$

$x^2 + (y-2)^2 = 4$ circle: center: $(0, 2)$ Radius: 2

#67 $r = \dfrac{2}{1-\sin\theta}$ $(1-\sin\theta)$ $(1-\sin\theta)$

$r - r\sin\theta = 2 \Rightarrow (r)^2 = (2 + r\sin\theta)^2$

$r - \underbrace{r\sin\theta}_{y} = 2$, $r = \sqrt{x^2+y^2}$

$r^2 = (2 + r\sin\theta)^2$

$x^2 + y^2 = (2 + y)^2$

$x^2 + y^2 = 4 + 4y + y^2$

$-y^2 \quad -4y \quad -y^2$

$x^2 - 4y = 4$ or $x^2 = 4 + 4y$ — both correct

Rectangular to polar

$x = r\cos\theta$, $y = r\sin\theta$, $x^2 + y^2 = r^2$, $\tan\theta = \frac{y}{x}$

Goal: try to solve for r

$x + y = 3$

$r\cos\theta + r\sin\theta = 3$

$\dfrac{r(\cos\theta + \sin\theta)}{\cos\theta + \sin\theta} = \dfrac{3}{\cos\theta + \sin\theta}$

$r = \dfrac{3}{\cos\theta + \sin\theta}$

Ex: $\frac{y}{x} = \frac{-x}{x}$ $\qquad \frac{y}{x} = -1$

$$\tan\theta = -1$$

$$\theta = -\pi/4 \quad \text{or} \quad \theta = \frac{3\pi}{4}$$

$$\theta = 7\pi/4$$

#79 $x^2 + (y-1)^2 = 1$ ⟶

$$(r\cos\theta)^2 + (r\sin\theta - 1)^2 = 1$$

$$r^2\cos^2\theta + r^2\sin^2\theta - 2r\sin\theta + 1 = 1$$

$$r^2(\underbrace{\cos^2\theta + \sin^2\theta}_{=1}) - 2r\sin\theta = 0$$

$$r^2 - 2r\sin\theta = 0$$

$$x^2 + y^2 - 2y + 1 = 1$$

$$r^2 - 2r\sin\theta = 0$$

$$\frac{r^2}{r} = \frac{2r\sin\theta}{r}$$

$$r = 2\sin\theta$$

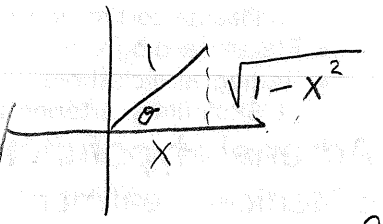
$\tan(\arccos x)$

$\cos\theta = \frac{\text{adj}}{\text{hyp}}$

$\tan\theta = \frac{\text{opp}}{\text{adj}}$

$\cos\theta = \frac{x \leftarrow \text{adj}}{1 \leftarrow \text{hyp}}$

$= \frac{\sqrt{1-x^2}}{x}$



$\tan(\arccos x) = \left|\frac{\sqrt{1-x^2}}{x}\right|$

$x^2 + p^2 = 1^2$

$-x^2 \qquad -x^2$

$p^2 = 1 - x^2$

$p = \sqrt{1-x^2}$

~~Not~~

$\sqrt{1-x^2}$