12/5/12 - Sec 6.3
Math 1060

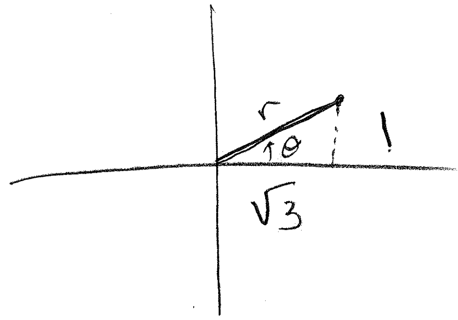
$a+bi \to (a, b)$

68 $\quad \overset{a}{\downarrow}\ \ \overset{b}{\swarrow}$

$z = \sqrt{3} + 1i$

$(\sqrt{3}, 1)$

z^4



$r^2 = \sqrt{3}^2 + 1^2$

$r^2 = 3 + 1$

$r^2 = 4$

$r = 2$

$z = 2(\cos 30^\circ + i \sin 30^\circ)$

$z \cdot z = 2 \cdot 2(\cos(30+30) + i \sin(30+30))$

$\tan\theta = \frac{b}{a}$

$z^2 = 4(\cos 60 + i \sin 60)$

$r = \sqrt{a^2+b^2}$

$z^3 = z^2 \cdot z$

$= 4 \cdot 2(\cos(60+30) + i \sin(60+30))$

$z^3 = 8(\cos 90 + i \sin 90)$

$z^4 = z^3 \cdot z$ or $z^2 \cdot z^2$

$= 4 \cdot 4(\cos(60+60) + i \sin(60+60))$

$z^4 = 16(\cos 120 + i \sin 120)$

$16\left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)$

$\boxed{-8 + 8\sqrt{3}\, i}$

Sec 6.3 Powers and Roots of Complex Numbers

#68 z^4

$$z = 2(\cos 30^\circ + i \sin 30^\circ)$$

$$z^2 = 4(\cos 60 + i \sin 60)$$

$$z^3 = 8(\cos 90 + i \sin 90)$$

$$z^4 = 16(\cos 120 + i \sin 120)$$

$$z^5 = \underset{2^5}{32}(\cos 150 + i \sin 150)$$

$$z^n = 2^n(\cos(n(30)) + i \sin(n(30)))$$

V R

De Moivre Theorem

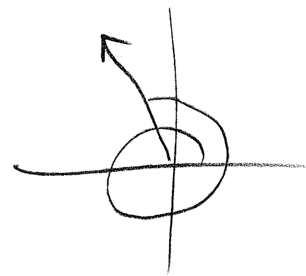
if $z = r(\cos\theta + i \sin\theta)$ is a complex number, and n is any positive integer, then

$$z^n = r^n(\cos n\theta + i \sin n\theta)$$

#3 $\left[\sqrt{2}\left(\cos 120^\circ + i \sin 120\right)\right]^4$

$= \left(\sqrt{2}\right)^4\left(\cos 120\cdot 4 + i \sin 120\cdot 4\right)$

$= 4\left(\cos 480 + i \sin 480\right)$

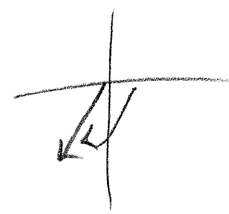


$4\left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)$

$\boxed{-2 + 2\sqrt{3}\, i}$

#12 $(1-i)^3 = -2-2i$

$\left[\sqrt{2}\left(\cos -45^\circ + i \sin -45^\circ\right)\right]^3$



$\left(\sqrt{2}\right)^3\left(\cos 3(-45) + i \sin 3(-45)\right)$

$2\sqrt{2}\left(\cos -135 + i \sin -135\right)$

$2\sqrt{2}\left(-\frac{\sqrt{2}}{2} + -\frac{\sqrt{2}}{2} i\right)$

$\frac{-\cancel{2}\sqrt{2}\sqrt{2}}{\cancel{2}} + -\frac{\cancel{2}\sqrt{2}\sqrt{2}}{\cancel{2}} i$

$\boxed{-2-2i}$

Roots of Complex Numbers

$\sqrt{1+i}$

square Roots of 9 $\qquad$ $x^3 = 1$

$\sqrt{9}$

Theorem: For any positive integer n, the complex Number

$$z = r(\cos\theta + i\sin\theta)$$

has exactly n distinct Roots given by the expression

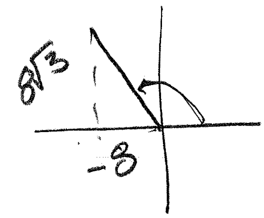
$$r^{1/n}\left[\cos\left(\frac{\theta+2\pi k}{n}\right) + i\sin\left(\frac{\theta+2\pi k}{n}\right)\right]$$

for $k = 0, 1, 2, \ldots, n-1$

$r^{1/n} = \sqrt[n]{r}$

Find the 4^{th} Roots of $-8+8i\sqrt{3}$

$16(\cos 120° + i \sin 120°)$



$\sqrt[4]{16} = 2$

4^{th} Roots:

$k=0 \quad 2\left(\cos \frac{120+360\cdot 0}{4} + i \sin \frac{120+360\cdot 0}{4}\right)$

$k=1 \quad 2\left(\cos \frac{120+360\cdot 1}{4} + i \sin \frac{120+360\cdot 1}{4}\right)$

$k=2 \quad 2\left(\cos \frac{120+360\cdot 2}{4} + i \sin \frac{120+360\cdot 2}{4}\right)$

$k=3 \quad 2\left(\cos \frac{120+360\cdot 3}{4} + i \sin \frac{120+360\cdot 3}{4}\right)$

$r^2 = (-8)^2 + (8\sqrt{3})^2$

$= 64 + 64\cdot 3$

$r^2 = 256$

$r = 16$

$\tan\theta = \frac{8\sqrt{3}}{-8}$

$\tan\theta = -\sqrt{3}$

$\theta = 120$

$k=0$

$2(\cos 30° + i \sin 30°) = 2\left(\frac{\sqrt{3}}{2} + i\frac{1}{2}\right)$.

$k=1$

$2(\cos 120 + i \sin 120) = 2\left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)$.

$k=2$

$2(\cos 210 + i \sin 210) = 2\left(-\frac{\sqrt{3}}{2} + i\,-\frac{1}{2}\right)$.

$k=3$

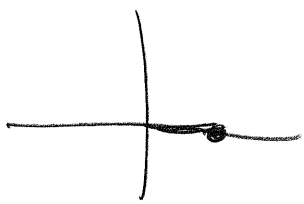
$2(\cos 300 + i \sin 300) = 2\left(\frac{1}{2} + i\,-\frac{\sqrt{3}}{2}\right)$

$\sqrt{3}+i, \quad -1+\sqrt{3}\,i, \quad -\sqrt{3}+-i, \quad 1-\sqrt{3}\,i$

$x^3 = 1$

The Roots of Unity are the various Roots of 1

1st trig form



$1 = 1(\cos 0° + i \sin 0°)$