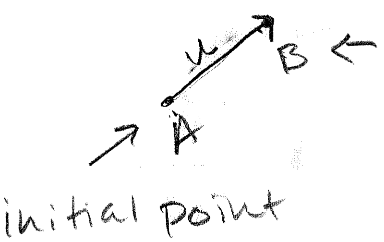
11/16/2012 - Sec 5.4 Math 1060

Sec 5.4 Vectors

Defn

A vector is a Quantitity with a magnitude and a direction often Represented by a directed line segment with length corresponding to the magnitude

Ex: Velocity, Acceleration, force



Name: $\overrightarrow{AB}$

or $\vec{u}$

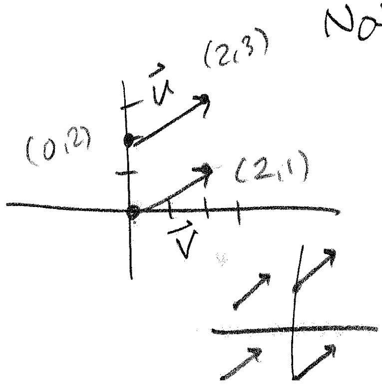
$\overrightarrow{BA}$

Two vectors are equal if

1. Same direction
2. Same magnitude

Notation: $|\overrightarrow{BA}| = |\vec{u}|$

or $\|\overrightarrow{AB}\|$



$\vec{u} = \vec{v}$

$$|\vec{u}| = \sqrt{(2-0)^2 + (3-2)^2} = \sqrt{4+1} = \sqrt{5}$$

$$|\vec{v}| = \sqrt{(2-0)^2 + (1-0)^2} = \sqrt{5}$$

Scalar Multiplication

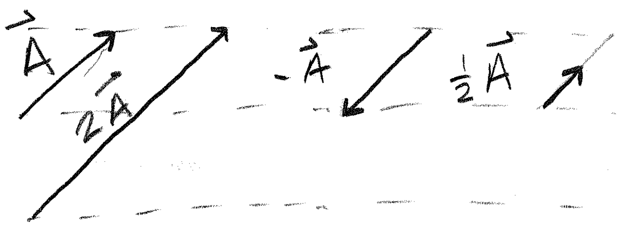
For any scalar k and $\vec{A}$, $k\vec{A}$ is a vector with magnitude $|k|$ times the magnitude of A

if k is positive $k\vec{A}$ is in the same direction

if k is Negative $k\vec{A}$ is in the opposite direction

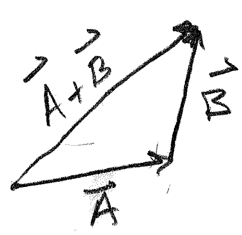
if k is 0 creates $\vec{0}$

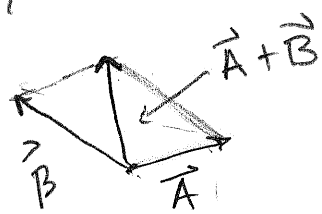
Ex

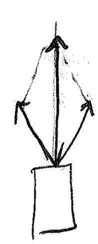


Vector Addition

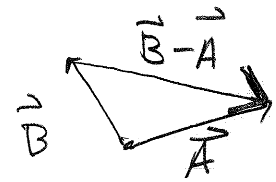
To find the sum (or Resultant) $\vec{A}+\vec{B}$ of any vectors $\vec{A}$ & $\vec{B}$, position $\vec{B}$ (without changing magnitude or direction) so that the initial of $\vec{B}$ coincides with the initial point of $\vec{A}$







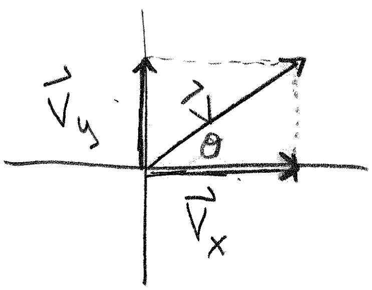
Vector Subtraction



$\vec{B} - \vec{A}$

Horizontal & Vertical Component

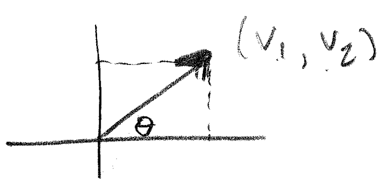
place vector vector at coordinate system initial at (0,0) called Standard position.



θ the direction angle and $\vec{V}$ is the sum of a vertical and Horizontal Vector

Component Form

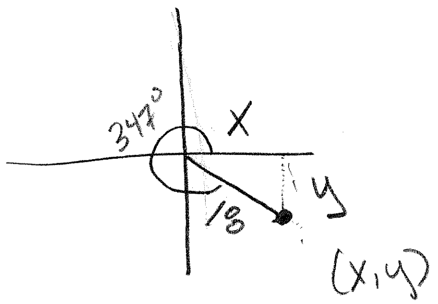
If the vector is in standard position with terminal point at (V_1, V_2) then $\vec{V} = \langle V_1 , V_2 \rangle$



$|\vec{V}| = \sqrt{V_1^2 + V_2^2}$

Find component form of a vector in standard position with

#35 $|v| = 18$ $\theta = 347°$



$18 \cdot \sin 347° = \frac{y}{18} \cdot 18$

$-4.0 = y$

$18 \cos 347° = \frac{x}{18} \cdot 18$

$17.5 = x$

$\langle 17.5, -4.0 \rangle$

With Component Form

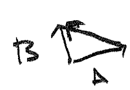
$\vec{A} = \langle a_1, a_2 \rangle$ $\vec{B} = \langle b_1, b_2 \rangle$ and k be a scalar

1. Scalar Product : Geometrically adjusted Magnitude by factor k

$k\vec{A} = \langle ka_1, ka_2 \rangle$

2. Vector Sum : 

$\vec{A} + \vec{B} = \langle a_1 + b_1, a_2 + b_2 \rangle$

3. Vector Difference : 

$\vec{A} - \vec{B} = \langle a_1 - b_1, a_2 - b_2 \rangle$

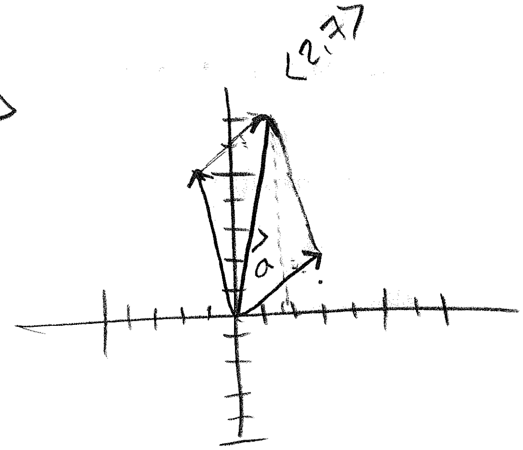
New operation!

4. Dot Product: Result is a Number
<u>Not</u> another vector

$$\vec{A} \cdot \vec{B} = a_1 b_1 + a_2 b_2$$

Ex:

$\vec{a} = \langle 3, 2 \rangle \qquad \vec{b} = \langle -1, 5 \rangle$



$\vec{a} + \vec{b} = \langle 3 + -1 \,,\, 2 + 5 \rangle$

$= \langle 2, 7 \rangle$

$2\vec{a} - \vec{b} \quad = \langle 6 - -1 \,,\, 4 - 5 \rangle$

$= \langle 7, -1 \rangle$

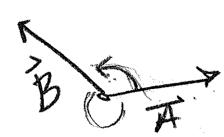
$2\langle 3, 2 \rangle$

$\langle 6, 4 \rangle - \langle -1, 5 \rangle$

$\vec{a} \cdot \vec{b} = 3(-1) + 2(5) = -3 + 10 = \boxed{7}$

$\langle 3, 2 \rangle \cdot \langle -1, 5 \rangle$

<u>The Angle in between two Vectors</u>



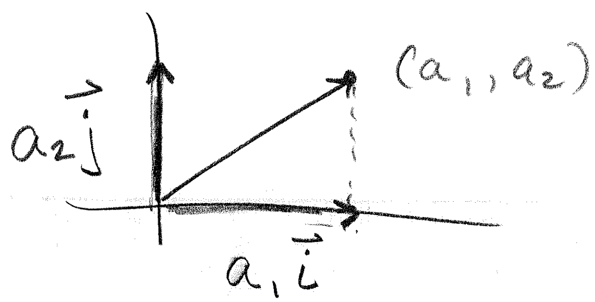
If $\vec{A}$ and $\vec{B}$ are Non zero vectors and α is the smallest angle between them. then

$$\cos \alpha = \frac{\vec{A} \cdot \vec{B}}{|\vec{A}||\vec{B}|}$$

Unit Vectors

The vectors $\vec{i} = \langle 1, 0 \rangle$ and $\vec{j} = \langle 0, 1 \rangle$ are called unit vectors.

for any vector $\langle a_1, a_2 \rangle = a_1\vec{i} + a_2\vec{j}$



$\vec{A} = \langle -2, 6 \rangle = -2\vec{i} + 6\vec{j}$