11/5/12 - Sec 5.1 Math 1060

#32 $\tan^2\theta - \cot^2\theta = 0$

$\sin^2\theta \cos^2\theta$ $\cancel{\sin^2 x}\cos^2 x$ $\sin^2 x \cos^2 x$

$$\frac{\sin^2\theta}{\cancel{\cos^2\theta}} - \frac{\cos^2\theta}{\cancel{\sin^2\theta}} = 0$$

$$\sin^4\theta - \cos^4\theta = 0$$

$$(\sin^2\theta - \cos^2\theta)(\underbrace{\sin^2\theta + \cos^2\theta}_{=1}) = 0$$

$$\sin^2\theta - \cos^2\theta = 0$$

$$\sin^2\theta - (1 - \sin^2\theta) = 0$$

$$\sin^2\theta - 1 + \sin^2\theta = 0$$

$$2\sin^2\theta - 1 = 0$$
$$+1 \quad +1$$

$$\frac{2\sin^2\theta}{2} = \frac{1}{2}$$

$$\sqrt{\sin^2\theta} = \sqrt{\frac{1}{2}}$$

$$\sin\theta = \pm\frac{1}{\sqrt{2}}$$

$$\sin\theta = \pm\frac{\sqrt{2}}{2}$$

Check

$\tan^2\theta - \cot^2\theta = 0$

$45°$

$1^2 - 1^2 = 0$ ✓

$135°$

$(-1)^2 - (-1)^2 = 0$ ✓

$225°$

$1^2 - 1^2 = 0$ ✓

$315°$

$(-1)^2 - (-1)^2 = 0$ ✓

$\theta = 45° + 360°k$, $135° + 360°k$

$\theta = -45° + 360°k$, $225° + 360°k$

$\boxed{\{45, 135°, 225°, 315°\}}$

Alt: way

$$\tan^2 \cdot \tan^2\theta - \frac{\overset{\tan^2}{1}}{\tan^2\theta} = \overset{\tan^2}{0}$$

$$\tan^4\theta - 1 = 0$$
$$\quad +1 \quad +1$$

$$\sqrt[4]{\tan^4\theta} = \sqrt[4]{1}$$

$$\tan\theta = \pm 1$$

#14

$$6\sin^2 x - 2\cos x = 5$$

$$6(1-\cos^2 x) - 2\cos x = 5$$

$$6 - 6\cos^2 x - 2\cos x = 5$$
$$-5 \qquad\qquad -5$$

$$\frac{-6\cos^2 x}{-1} - \frac{2\cos x}{-1} + \frac{1}{-1} = \frac{0}{-1}$$

$$6\cos^2 x + 2\cos x - 1 = 0$$
$$a = 6 \quad b = 2 \quad c = -1$$

$$\cos x = \frac{-2 \pm \sqrt{4 - 4(6)(-1)}}{2(6)}$$

$$= \frac{-2 \pm \sqrt{4 + 24}}{12}$$

$$\cos x = \frac{-2 \pm \sqrt{28}}{12}$$

$(-2+\sqrt{28})/12$ $\qquad\qquad$ $(-2-\sqrt{28})/12$

$\cos x = .27429$ $\qquad\qquad$ $\cos x = -.60763$

$x = \cos^{-1}(.27429)$ $\qquad\qquad$ $x = \cos^{-1}(-.60763)$

$x = 1.2929 + 2\pi k$ $\qquad\qquad$ $x = 2.2238 + 2\pi k$

$\;\;= 4.9903 + 2\pi k$ $\qquad\qquad$ $\;\;= 4.0954 + 2\pi k$

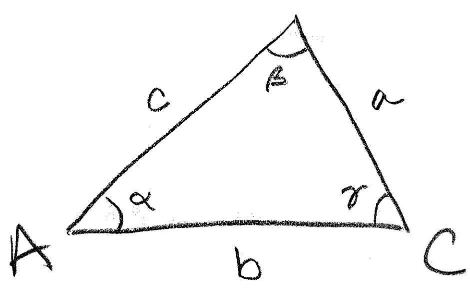
$2\pi - 1.2929$ $\qquad\qquad$ $2\pi - 2.2238$

$\{1.2929,\ 2.2238,\ 4.9903,\ 4.0954\}$

$\{1.3,\ 2.2,\ 5.0,\ 4.1\}$

Sec 5.1 The Law of Sines

<u>Oblique Triangle</u>: a triangle with No Right angle

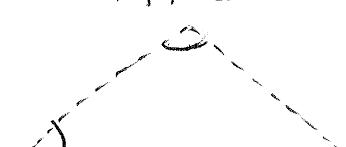


Gamma γ

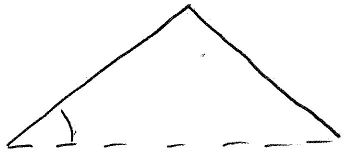
1. one side & 2 angles (ASA, AAS)

ASA

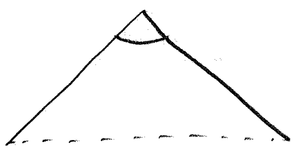
AAS



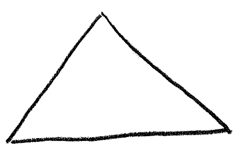
2. two sides and 1 Non included angle (SSA)



3. Two sides and an included Angle (SAS)



4. All three sides (SSS)



5. 3 Angles (AAA) ← similar triangles

We must know <u>atleast</u> 1 side

The Law of Sines : A Relationship between the Sines of the angles and the sides of the triangle

$$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c} \quad \text{or} \quad \frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$$

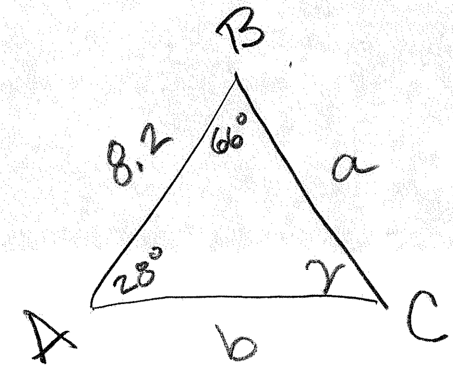
To use Law of Sines I must know one side and the angle opposite

Solve triangle

Beta ↙ Lower case ↙

$\alpha = 28^\circ$, $\beta = 66^\circ$, and $c = 8.2$

B ← Bee



$$28 + 66 + \gamma = 180$$

$$94 + \gamma = 180$$

$$-94 \qquad -94$$

$$\gamma = 86^\circ \checkmark$$

$$\frac{c}{\sin\gamma} = \frac{b}{\sin\beta}$$

$$\frac{8.2 \cdot \sin 66}{\sin 86^\circ} = \frac{b}{\sin 66^\circ} \cdot \sin 66$$

$$b = \frac{8.2 \sin 66}{\sin 86}$$

$$b = 7.5094$$

$$\frac{a}{\sin\alpha} = \frac{c}{\sin\gamma}$$

$$\sin 28 \cdot \frac{a}{\sin 28} = \frac{8.2}{\sin 86} \cdot \sin 28$$

$$a = \frac{8.2 \sin 28}{\sin 86}$$

$$= 3.8591$$

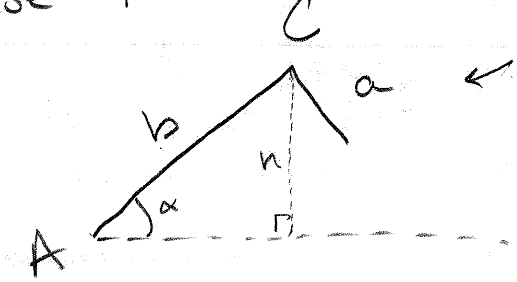
Round to tenth

$\gamma = 86^\circ$, $b = 7.5$, $a = 3.9$

Possible Cases for SSA

if Angle is acute

Case 1



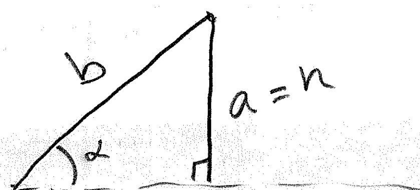
doesn't reach
so no triangle

No triangle
if $a < h$

$$b \cdot \sin\alpha = \frac{h}{b} \cdot b$$

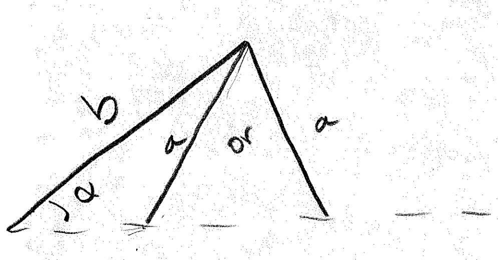
$$h = b \cdot \sin\alpha$$

Case 2



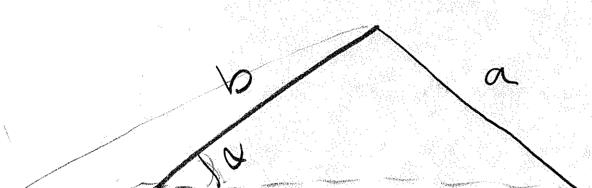
exactly 1 triangle
if $a = h$

Case 3



creates 2 triangles
if $a > h$
and $a < b$

Case 4



creates 1 triangle
if $a > h$
and $a > b$