Sec 4.1 Cont.

$\sin^{-1}(x) \leftarrow$ Quad I, IV

$\cos^{-1}(x) \leftarrow$ Quad I, II

$\tan^{-1}(x) \leftarrow$ Quad I, IV

Inverse cosecant

$\csc^{-1}(x)$ or $\operatorname{arccsc}(x)$

$$\csc\theta = \frac{1}{\sin\theta}$$

$$\underbrace{\csc^{-1}(x)}_{\text{Ratio}} = \underbrace{\sin^{-1}\left(\frac{1}{x}\right)}_{\text{Reciprocal Ratio}}$$
, Returns angles in Quad I, IV

Since $\sec\theta = \frac{1}{\cos\theta}$, $\sec^{-1}(x) = \cos^{-1}\left(\frac{1}{x}\right)$

Returns angles in Quad I, II

Since $\cot\theta = \frac{1}{\tan\theta}$

$\cot^{-1}(x) = \tan^{-1}\left(\frac{1}{x}\right)$ Returns an angle in Quad I, II

Return angle in Quad II

Return angle in Quad IV

$$\cot^{-1}\left(\frac{-1}{}\right) = \tan^{-1}\left(\frac{1}{-1}\right)$$

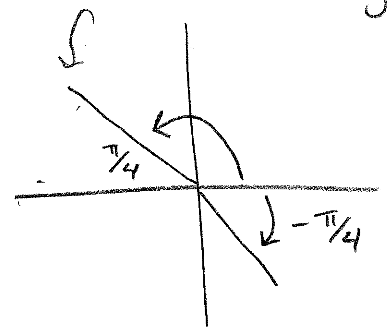
$$= \tan^{-1}(-1)$$

$$= -\frac{\pi}{4}$$

$$= \pi - \frac{\pi}{4}$$

$$= \frac{3\pi}{4}$$

π - Ref Angle



$$\cot^{-1}(x) = \begin{cases} \tan^{-1}\left(\frac{1}{x}\right) & x > 0 \\ \pi + \tan^{-1}\left(\frac{1}{x}\right) & x < 0 \\ \pi/2 & x = 0. \end{cases}$$

$\cot^{-1}(0)$

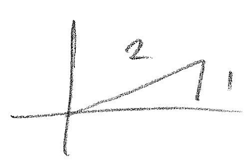
$$\cot\theta = 0$$

$$\frac{\cos\theta}{\sin\theta} = 0$$

$$\theta = \pi/2 \checkmark$$

$$\csc^{-1}(2) = \underbrace{\sin^{-1}\left(\frac{1}{2}\right)}_{} \leftarrow 30^\circ$$

$$= \frac{\pi}{6}$$



$\sec^{-1}(6) = \cos^{-1}\left(\frac{1}{6}\right) = 1.4033$

Special Ratios

$1/2$

$\sqrt{3}/2$

$\sqrt{2}/2$

$\pm 1, 0$

Composition of functions

$(f\circ g)(x) = 3(5x-7)+2$

$f(x) = 3x+2$

$g(x) = 5x-7$ $\quad g(2) = 5\cdot 2 - 7 = 3$

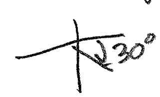
$(f\circ g)(2) = f(g(2)) = f(3) = 3\cdot 3+2$

Ex: $\underbrace{\arctan(\underbrace{\tan(\overset{\text{angle}}{60^\circ})}_{\text{Ratio}})}_{\text{angle}} = 60^\circ$

$\underbrace{\cos(\underbrace{\cos^{-1}(\overset{\text{Ratio}}{1/2})}_{\text{angle}})}_{\text{Ratio}} = 1/2$

Ex: 30° Angle in Quad III

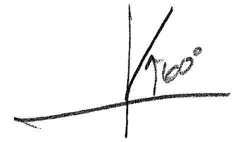
$\underbrace{\arcsin(\underbrace{\sin(210^\circ)}_{-\text{Ratio}})}_{\text{Angle in IV}}$



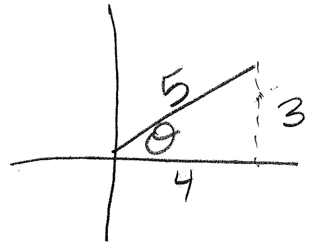
$= -30^\circ$

$\underbrace{\arcsin(\underbrace{\sin(\overset{\text{angle in Quad II}}{120^\circ})}_{+\text{ Ratio}})}_{\text{angle in Quad I}} \neq 120^\circ$

$= 60^\circ$



$$\tan\Big(\underbrace{\arcsin\left(\tfrac{3}{5}\right)}_{\theta}\Big)$$



$$\tan\theta = \frac{\text{opp}}{\text{adj}}$$

$$= \frac{3}{4}$$

$\sin\theta = 3/5$

$x^2 + 3^2 = 5^2$

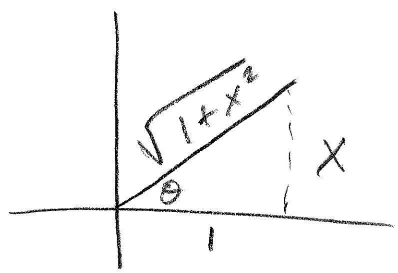
$x^2 + 9 = 25$

$x^2 = 16$

$x = \pm 4$

#93 $\cos\Big(\underbrace{\arctan(x)}_{\theta}\Big)$

$$\cos\theta = \boxed{\frac{1}{\sqrt{1+x^2}}}$$



$$\cos(\arctan(x)) = \frac{1}{\sqrt{1+x^2}}$$

$\tan\theta = x = \frac{x}{1}$ ← opp, ← adj

$1^2 + x^2 = h^2$

$\sqrt{1+x^2} = h$

#92 $\cos\Big(\overbrace{\arcsin(x)}^{\theta}\Big)$

votes:

$\sqrt{1-x^2}$

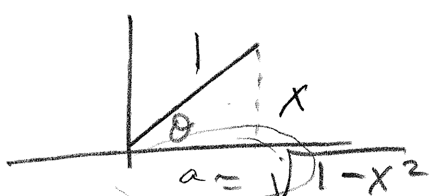
$\sqrt{x^2-1}$

$\sqrt{x^2+a^2}$

$\cos\theta$

$= \frac{\sqrt{1-x^2}}{1}$

$= \sqrt{1-x^2}$



$\sin\theta = \frac{x}{1}$

$a^2 + x^2 = 1$

$a^2 = 1 - x^2$

$a = \sqrt{1-x^2}$