

#54

10/22/2012 - Sec 4.1

Math 1060

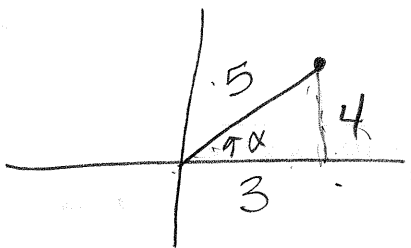
$$\begin{aligned}
 \sin^2 x - \sin^2 y &= \sin(x+y) \sin(x-y) \\
 &= (\sin x \cos y + \cos x \sin y) (\sin x \cos y - \cos x \sin y) \\
 &= \sin^2 x \cos^2 y - \sin x \cos y \cos x \sin y \\
 &\quad + \sin x \cos y \cos x \sin y - \cos^2 x \sin^2 y \\
 &= \sin^2 x \cos^2 y - \cos^2 x \sin^2 y \\
 &= \sin^2 x (1 - \sin^2 y) - (1 - \sin^2 x) \sin^2 y \\
 &= \sin^2 x - \sin^2 x \sin^2 y - \sin^2 y + \sin^2 x \sin^2 y \\
 &= \sin^2 x - \sin^2 y \quad \checkmark
 \end{aligned}$$

$$\# 26 \quad \cos\left(\frac{x}{2}\right) + \cos\left(\frac{y}{3}\right)$$

$$\begin{aligned}
 \cos x + \cos y &= 2 \cos\left(\frac{x+y}{2}\right) \cos\left(\frac{x-y}{2}\right) \\
 &= 2 \cos\left(\frac{\frac{1}{2} + \frac{2}{3}}{2}\right) \cos\left(\frac{\frac{1}{2} - \frac{2}{3}}{2}\right) \left\{ \begin{array}{l} 3 \cdot \frac{1}{2} + \frac{2}{3} \cdot 2 \\ 3 \cdot \frac{1}{2} - \frac{2}{3} \cdot 2 \end{array} \right. \\
 &= 2 \cos\left(\frac{7}{12}\right) \cos\left(-\frac{1}{12}\right) \\
 &\quad \left. \begin{array}{l} \frac{3+4}{6} = 7/6 \\ 7/6 \cdot \frac{1}{2} = 7/12 \end{array} \right\}
 \end{aligned}$$

$$\boxed{2 \cos\left(\frac{7}{12}\right) \cos\left(\frac{1}{12}\right)}$$

$$\cos \alpha = \frac{3}{5}, \text{ Quad I}$$



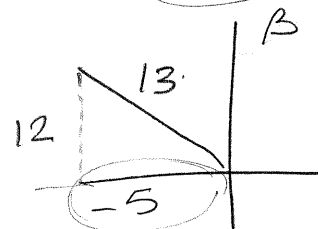
$$\begin{aligned} \text{OPP}^2 + 3^2 &= 5^2 \\ \text{OPP}^2 + 9 &= 25 \\ \sqrt{\text{OPP}^2} &= \sqrt{16} \\ \text{OPP} &= 4 \end{aligned}$$

$$\sin(\alpha + \beta) \text{ Given } \cos \alpha = \frac{3}{5}, \text{ Quad I}, \sin \beta = \frac{12}{13}, \text{ Quad II}$$

$$\sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\left(\frac{4}{5}\right) \left(-\frac{5}{13}\right) + \left(\frac{3}{5}\right) \left(\frac{12}{13}\right)$$

$$-\frac{20}{65} + \frac{36}{65} = \frac{16}{65}$$



$$\sin\left(\frac{x}{2}\right) =$$

Example Conditional Equation

$$\begin{array}{r} 3x + 5 = 14 \\ -5 \quad -5 \end{array} \quad x = 3$$

$$\begin{array}{r} 3x = 9 \\ \frac{3}{3} \quad \frac{9}{3} \\ x = 3 \end{array}$$

See 4.1 Inverse Trig Functions

$$\sqrt{x^2} = \sqrt{16}$$

$$x = \pm 4$$

INVERSES  
[square root function  
[squaring function

$$(\sqrt{x})^2 = (2)^2$$

$$x = 4$$

$$\sin x = \frac{4}{5}$$

Remember

Trig function (angle) = Ratio

Inverse Trig function (Ratio) = angle

Defn:

For  $-1 \leq x \leq 1$ ,  $y = \sin^{-1}(x)$  provided

$$\sin(y) = x \quad \text{and} \quad -\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$$

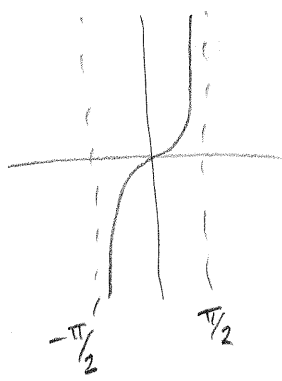
(angles in Quad I and IV)

For  $-1 \leq x \leq 1$ ,  $y = \cos^{-1}(x)$  provided

$$\cos(y) = x \quad \text{and} \quad 0 \leq y \leq \pi$$

(angles in Quad I and II)

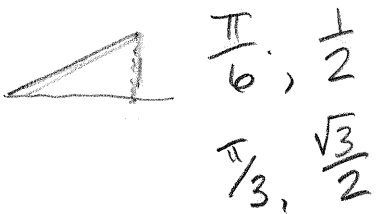
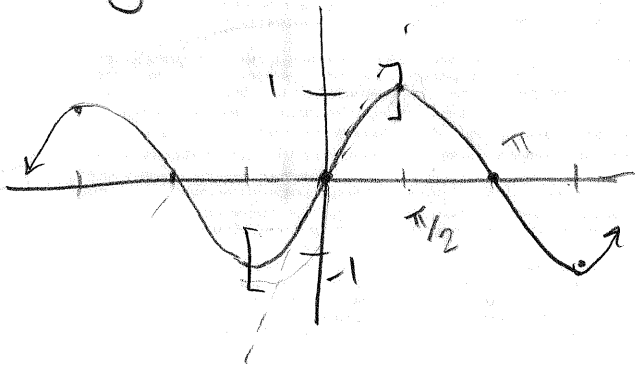
$\tan \theta$



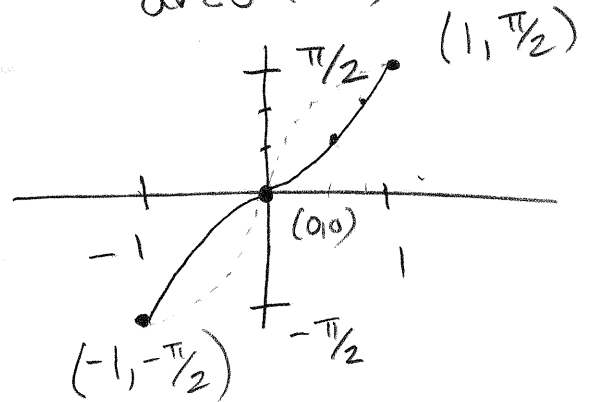
For any Real Number  $x$ ,  $y = \tan^{-1}(x)$   
 provided  $x = \tan(y)$  and  $-\frac{\pi}{2} < y < \frac{\pi}{2}$   
 (angles in Quad I & IV)

### Graphs

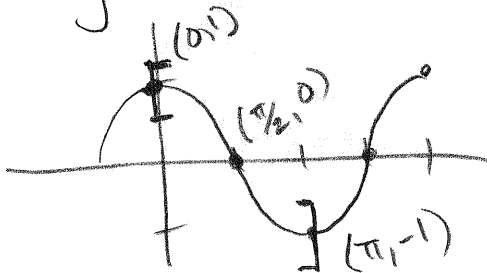
$y = \sin x$



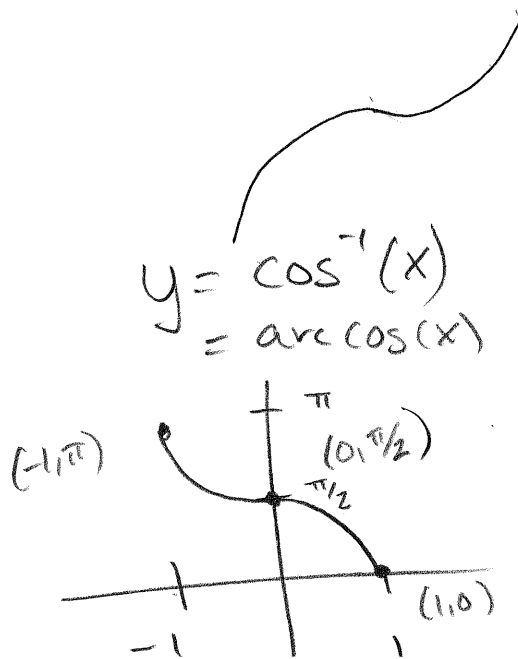
$y = \sin^{-1}(x)$   
 $= \arcsin(x)$



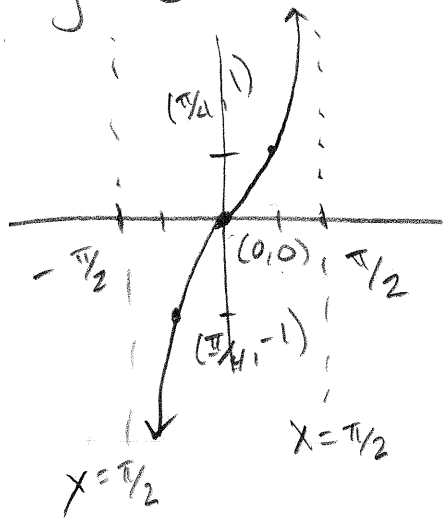
$y = \cos x$



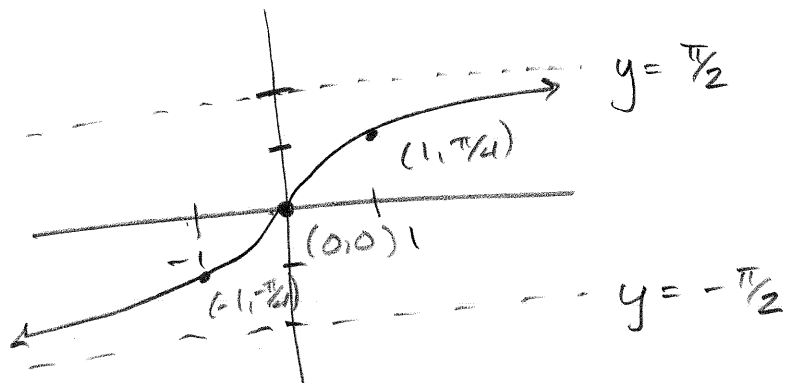
$y = \cos^{-1}(x)$   
 $= \arccos(x)$



$$y = \tan x$$



$$y = \tan^{-1}(x) = \arctan(x)$$



Evaluate in degrees ←

$$\sin^{-1}\left(\frac{\sqrt{2}}{2}\right) = 45^\circ$$

$$\begin{aligned} \tan^{-1}(-\sqrt{3}) &= -\frac{\pi}{3} \\ &= -60^\circ \end{aligned}$$

$$\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right) = 135^\circ$$

Note if not specifically stated whether want degrees or Radians, answer in Radians

Ref:  $45^\circ$

