10/17/2012 - Sec 3.6 Math 1060

# Sec 3.6 Product and Sum Identities

## Product to Sum

$$\sin A \cos B$$

$$\sin(A+B) = \sin A \cos B + \cancel{\cos A \sin B}$$

$$\sin(A-B) = \sin A \cos B - \cancel{\cos A \sin B}$$

$$\frac{\sin(A+B) + \sin(A-B)}{2} = \frac{2 \sin A \cos B}{2}$$

Given

$$\sin A \cos B = \frac{1}{2}\left[\sin(A+B) + \sin(A-B)\right]$$

$$\sin A \sin B = \frac{1}{2}\left[\cos(A-B) - \cos(A+B)\right]$$

$$\cos A \sin B = \frac{1}{2}\left[\sin(A+B) - \sin(A-B)\right]$$

$$\cos A \cos B = \frac{1}{2}\left[\cos(A-B) + \cos(A+B)\right]$$

#8 $\cos(3t)\sin(5t)$

$$\frac{1}{2}\left[\sin(3t+5t) - \sin(3t-5t)\right]$$

$$\frac{1}{2}\left[\sin(8t) - \sin(-2t)\right]$$

$$\frac{1}{2}\left[\sin(8t) + \sin(2t)\right]$$

#11 Find the exact value of

$$\overset{A}{\sin(52.5^\circ)}\ \overset{B}{\sin(7.5^\circ)}$$

$52.5^\circ + 7.5^\circ = 60^\circ$

$52.5^\circ - 7.5^\circ = 45^\circ$

$$\sin A \sin B = \frac{1}{2}\left[\cos(A-B) - \cos(A+B)\right]$$

$$\frac{1}{2}\left[\cos(52.5-7.5) - \cos(52.5+7.5)\right]$$

$$\frac{1}{2}\left(\cos 45 - \cos 60\right)$$

$$\frac{1}{2}\left(\frac{\sqrt{2}}{2} - \frac{1}{2}\right)$$

$$\frac{1}{2}\left(\frac{\sqrt{2}-1}{2}\right)$$

$$\frac{\sqrt{2}-1}{4}$$

## Sum-to-Product

$$\sin A \cos B = \frac{1}{2}\left[\sin(A+B) + \sin(A-B)\right]$$

Let $A+B=x$ and Let $A-B=y$

$$\begin{aligned} A + \cancel{B} &= x \\ A - \cancel{B} &= y \\ \hline \frac{2A}{2} &= \frac{x+y}{2} \end{aligned} \qquad \begin{aligned} \cancel{A} + B &= x \\ -\cancel{A} + B &= -y \\ \hline \frac{2B}{2} &= \frac{x-y}{2} \end{aligned}$$

$$A = \frac{x+y}{2} \qquad B = \frac{x-y}{2}$$

$$2\sin A\cos B = \sin(A+B) + \sin(A-B)$$

Given

$$\sin x + \sin y = 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

$$\sin x - \sin y = 2\cos\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$

$$\cos x + \cos y = 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

$$\cos x - \cos y = -2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$

#23 $\overset{x}{\sin(5\alpha)} - \overset{y}{\sin(8\alpha)}$

$$= 2\cos\left(\frac{5\alpha+8\alpha}{2}\right)\sin\left(\frac{5\alpha-8\alpha}{2}\right)$$

$$= 2\cos\left(\frac{13\alpha}{2}\right)\sin\left(\frac{-3\alpha}{2}\right)$$

$$= -2\cos\left(\frac{13\alpha}{2}\right)\sin\left(\frac{3\alpha}{2}\right)$$

#28 $\overset{x}{\sin(285)} - \overset{y}{\sin(15)}$

$$2\cos\left(\frac{285+15}{2}\right)\sin\left(\frac{285-15}{2}\right)$$

$$2\cos 150^\circ \sin 135^\circ$$

$$2\left(\frac{-\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$$

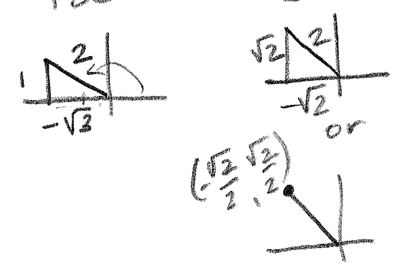
$$\frac{-\cancel{2}\sqrt{6}}{\cancel{4}2}$$

$$-\sqrt{6}/2$$

$$\frac{x+y}{2} = \frac{285+15}{2} = \frac{300}{2} = 150$$

$$\frac{x-y}{2} = \frac{285-15}{2} = \frac{270}{2} = 135$$

$150^\circ$ $135^\circ$



Given

## The Reduction Formula

If $\alpha$ is an angle in Standard Position whose terminal side contains $(a,b)$, then

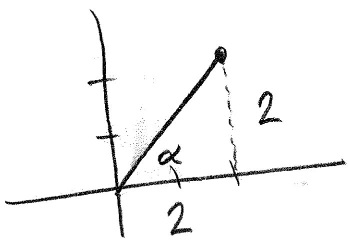
$$a\sin x + b\cos x = \sqrt{a^2+b^2}\,\sin(x+\alpha)$$

for any Real Number $x$,

Note: $\alpha$ Needs to be in Radians

#32 $\quad 2\sin x + 2\cos x$ Rewrite in form $A\sin(x+C)$

$\uparrow$ a $\qquad \uparrow$ b




$\tan\alpha = \frac{2}{2}$

$\tan\alpha = 1$

$\alpha = 45°$

$\alpha = \frac{\pi}{4}$

$$\boxed{2\sqrt{2}\,\sin\left(x+\frac{\pi}{4}\right)}$$

$\sqrt{a^2+b^2}$

$\sqrt{2^2+2^2}$

$\sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$

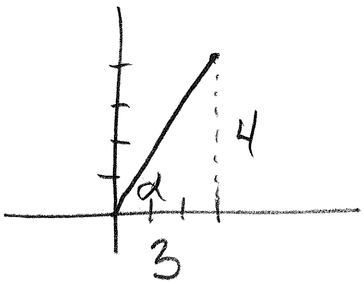
8

2 4

2 2

# 43

$$y = \underset{\substack{\uparrow \\ a}}{3} \sin x + \underset{\substack{\uparrow \\ b}}{4} \cos x$$




$\tan \alpha = \frac{4}{3}$

$\alpha = .93$

$\sqrt{a^2+b^2}$

$\sqrt{3^2+4^2}$

$\sqrt{9+16}$

$\sqrt{25} = 5$

$y = 5 \sin(x + .93)$

Amp $= 5$

phase shift $= -.93$

## Application

Modeling the motion of Spring

$$X = \frac{V_0}{\omega} \sin(\omega t) + X_0 \cos(\omega t)$$

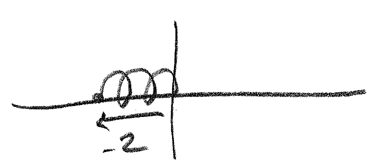
$t$ = time it's moving

$x$ = location of the end of spring

$V_0$ = initial velocity

$X_0$ = initial

$\omega$ = a constant




$X_0 = -2$

$V_0 = \underset{+2}{\longrightarrow}$

See Example 7