3.3 10/10/2012 – Sec 3.5 Math 1060

#86

$$\sec(v+t) = \frac{\cos v \cos t + \sin v \sin t}{\cos^2 v - \sin^2 t}$$

$$\frac{1}{\cos(v+t)}$$

$$\frac{1\,(\cos v \cos t + \sin v \sin t)}{(\cos v \cos t - \sin v \sin t)(\cos v \cos t + \sin v \sin t)}$$

$$\frac{\cos v \cos t + \sin v \sin t}{\cos^2 v \cos^2 t - \sin^2 v \sin^2 t}$$ ✓

$\sin^2 t + \cos^2 t = 1$

$-\sin^2 t$

$\cos^2 t = (1 - \sin^2 t)$

$\sin^2 v + \cos^2 v = 1$

$-\cos^2 v \quad -\cos^2$

$\sin^2 v = 1 - \cos^2 v$

$$\frac{\cos v \cos t + \sin v \sin t}{\cos^2 v (1 - \sin^2 t) - (1 - \cos^2 v) \sin^2 t}$$

same

$$\cos^2 v - \cancel{\cos^2 v \sin^2 t} - \sin^2 t + \cancel{\cos^2 v \sin^2 t}$$

$$\boxed{\frac{\cos v \cos t + \sin v \sin t}{\cos^2 v - \sin^2 t}}$$ ✓

Sec 3.4

#52

$$\frac{\cos(\alpha+\beta)}{\sin(\alpha-\beta)} = \frac{1-\tan\alpha\tan\beta}{\tan\alpha-\tan\beta}$$

$$\frac{\cos\alpha\cos\beta-\sin\alpha\sin\beta}{\sin\alpha\cos\beta-\cos\alpha\sin\beta}\cdot\frac{\frac{1}{\cos\alpha\cos\beta}}{\frac{1}{\cos\alpha\cos\beta}}$$

$$\left[\frac{\frac{\cancel{\cos\alpha}\,\cancel{\cos\beta}}{\cancel{\cos\alpha}\,\cancel{\cos\beta}}-\boxed{\frac{\sin\alpha}{\cos\alpha}}\boxed{\frac{\sin\beta}{\cos\beta}}}{\boxed{\frac{\sin\alpha}{\cos\alpha}}\frac{\cancel{\cos\beta}}{\cancel{\cos\beta}}-\frac{\cancel{\cos\alpha}}{\cancel{\cos\alpha}}\boxed{\frac{\sin\beta}{\cos\beta}}}\right]$$

$$\frac{1-\tan\alpha\tan\beta}{\tan\alpha-\tan\beta}\ \checkmark$$

sec 3.3

#62

$$\cos\left(y-\frac{\pi}{2}\right)\cos(y) + \sin\left(\frac{\pi}{2}-y\right)\sin(-y)$$

$$\cos\left[-\left(\frac{\pi}{2}-y\right)\right] \qquad - \cos(y)\sin(y)$$

$$\cos\left(\frac{\pi}{2}-y\right)$$

$$\sin(y)\cos(y) - \cos(y)\sin(y) = \boxed{0}$$

$$\cos\left(\frac{\pi}{2}-y\right) = \sin y$$

$$\cos\left(y-\frac{\pi}{2}\right) = \sin y$$

$$\sin\left(y-\frac{\pi}{2}\right)$$

$$\sin\left[-\left(\frac{\pi}{2}-y\right)\right]$$

$$-\sin\left(\frac{\pi}{2}-y\right) = -\cos y$$

#68 $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

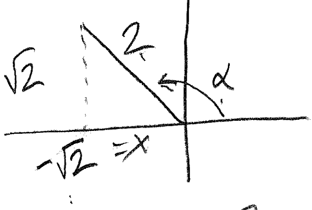
if $\sin\alpha = \frac{\sqrt{2}}{2}$ $\quad \sin\beta = \frac{1}{2}$

α in Quad II $\quad \beta$ in Quad I

$$= \left(\frac{-\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{2}}{2}\cdot\frac{1}{2}\right)$$

$$= \frac{-\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$$

$$\boxed{\frac{-\sqrt{6}-\sqrt{2}}{4}}$$



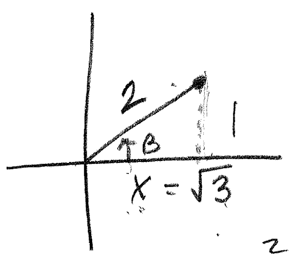
$$x^2 + (\sqrt{2})^2 = 2^2$$

$$x^2 + 2 = 4$$

$$\quad -2 \quad -2$$

$$\sqrt{x^2} = \sqrt{2}$$

$$x = \pm\sqrt{2}$$



$$x^2 + 1^2 = 2^2$$

$$x^2 + 1 = 4$$

$$\sqrt{x^2} = \sqrt{3}$$

$$x = \pm\sqrt{3}$$

Sec 3.5 Double - Angle Identities
half - Angle Identities

Double Angle Identities

* $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$$\begin{aligned}
\cos 2x &= \cos(x+x) \\
&= \cos x\cos x - \sin x \sin x \\
&= \cos^2 x - \sin^2 x \quad (1) \\
&= (1-\sin^2 x) - \sin^2 x \\
&= 1-\sin^2 x - \sin^2 x \\
&= 1-2\sin^2 x \quad (2) \\
&= \cos^2 x - (1-\cos^2 x) \\
&= \cos^2 x - 1 + \cos^2 x \\
&= 2\cos^2 x - 1 \quad (3)
\end{aligned}$$

$\sin^2 x + \cos^2 x = 1$

$-\sin^2 x \qquad -\sin^2$

$\cos^2 x = 1 - \sin^2$

$\sin^2 x = 1 - \cos^2 x$

$$\boxed{\begin{aligned}
\cos 2x &= \cos^2 x - \sin^2 x \\
&= 1 - 2\sin^2 x \\
&= 2\cos^2 x - 1
\end{aligned}}$$

$$\begin{aligned}
\sin 2x &= \sin(x+x) \\
&= \sin x\cos x + \cos x \sin x \\
&= \sin x \cos x + \sin x \cos x
\end{aligned}$$

* $\boxed{\sin 2x = 2\sin x\cos x}$

$$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$$

$$\tan(2x) = \tan(x+x)$$

$$= \frac{\tan x + \tan x}{1 - \tan x \tan x}$$

$$\boxed{\tan(2x) = \frac{2\tan x}{1-\tan^2 x}}$$

$\tan^2 x + 1 = \sec^2 x$

Ex: $\sin 90^\circ = 1$

$$\sin(2\cdot 45^\circ) = 2\sin 45^\circ \cos 45^\circ$$

$$= \cancel{2}^{1} \cdot \frac{\sqrt{2}}{\cancel{2}} \cdot \frac{\sqrt{2}}{2}$$

$$= \frac{\cancel{2}\cdot\cancel{2}}{\cancel{2}\cdot\cancel{2}}$$

$$= 1 \checkmark$$

#40 Verify

$$\cos(3t) = \cos^3 t - 3\sin^2 t \cos t$$

$$\cos(t+2t)$$

$$\cos(t)\cos(2t) - \sin(t)\sin(2t)$$

$$\cos(t)(\cos^2 t - \sin^2 t) - \sin(t)\, 2\sin(t)\cos(t)$$

$$\cos^3 t - \sin^2 t \cos t - 2\sin^2 t \cos t$$

$$\cos^3 t - 3\sin^2 t \cos t \checkmark$$

Half-angles

$$\cos 2\theta = 1 - 2\sin^2\theta$$

$$-1 \qquad\qquad -1$$

$$\frac{-1 + \cos 2\theta}{-2} = \frac{-2\sin^2\theta}{-2}$$

$$\sqrt{\frac{1 - \cos 2\theta}{2}} = \sqrt{\sin^2\theta}$$

$$\sin\theta = \pm\sqrt{\frac{1-\cos 2\theta}{2}}$$

Give * $\boxed{\sin\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1-\cos x}{2}}}$