10/5/2012 - Sec 3.3 Math 1060

#50 $(1+\cot\alpha)^2 - 2\cot\alpha = \dfrac{1}{(1-\cos\alpha)(1+\cos\alpha)}$

$1 + \cancel{2\cot\alpha} + \cot^2\alpha - \cancel{2\cot\alpha} =$

$1+\cot^2\alpha$

$\csc^2\alpha$

$\dfrac{1}{\sin^2\alpha}$

$\dfrac{1}{1-\cos^2\alpha}$

$\dfrac{1}{(1-\cos\alpha)(1+\cos\alpha)} =$

$\dfrac{1}{1-\cos^2\alpha}$

$\dfrac{1}{\sin^2\alpha}$

$\csc^2\alpha$

✓

#44 $\dfrac{\cos(x)\sin^2(x) + \cos^3(x)}{\sin(x)} = \cot(x)$

$\dfrac{\cos(x)\left(\underbrace{\sin^2 x + \cos^2 x}_{1}\right)}{\sin x}$

$\dfrac{\cos x}{\sin x}$

$\cot x$

#64

$$\frac{1-2\cos^2 y}{1-2\cos y\sin y} = \frac{(\sin y+\cos y)(\sin y-\cos y)}{(\sin y-\cos y)(\sin y-\cos y)}$$

$$\frac{(1-\cos^2 y)-\cos^2 y}{1-2\cos y\sin y}$$

$$\frac{(\sin y+\cos y)(\sin y-\cos y)}{\sin^2 x-2\cos y\sin y+\cos^2 x}$$

$$\frac{\sin^2 y-\cos^2 y}{1-2\cos y\sin y}$$

$$\frac{(\sin y+\cos y)(\sin y-\cos y)}{\sin^2 x+\cos^2 x-2\cos y\sin y}$$

$1=$

$1-\cos y\sin y-\cos y\sin y$

$$\frac{(\sin y+\cos y)(\sin y-\cos y)}{1-2\cos y\sin y}$$

$$\frac{(\sin y+\cos y)(\sin y-\cos y)}{1-\cos y\sin y-\cos y\sin y}$$

$$\frac{\sin^2 y-\cos^2 y}{1-2\cos y\sin y}$$

$$\frac{1-\cos^2 y-\cos^2 y}{1-2\cos y\sin y}$$

Circle Note

$-(\sin^2 x+\cos^2 x)=-(1)$

$-\sin^2 x-\cos^2 x=-1$

$$\frac{1-2\cos^2 y}{1-2\cos y\sin y}$$ ✓

72. $\ln(\tan\theta) = \ln(\sin\theta) + \ln(\sec\theta)$

$\ln\left(\frac{\sin\theta}{\cos\theta}\right)$

$r\log x$
$= \log x^r$

$\ln(\sin\theta) + {}^{-1}\ln(\cos\theta)$

$\ln(\sin\theta) + \ln\left[(\cos\theta)^{-1}\right]$

$\ln(\sin\theta) + \ln\left(\frac{1}{\cos\theta}\right)$

$\ln(\sin\theta) + \ln(\sec\theta)$

Way 2

$\ln\left(\sin\theta \cdot \frac{1}{\cos\theta}\right)$

$\ln(\sin\theta \cdot \sec\theta)$

$\ln(\sin\theta) + \ln(\sec\theta)$

#54 $\dfrac{\csc x}{\cot x} - \dfrac{\cot x}{\csc x} = \dfrac{\tan x}{\csc x}$

$$\frac{\frac{1}{\sin x}}{\frac{\cos x}{\sin x}} - \frac{\frac{\cos x}{\sin x}}{\frac{1}{\sin x}}$$

$$\frac{1}{\cancel{\sin x}} \cdot \frac{\cancel{\sin x}}{\cos x} - \frac{\cos x}{\cancel{\sin x}} \cdot \frac{\cancel{\sin x}}{1}$$

$$\frac{1}{\cos x} - \frac{\cos x \cdot \cos x}{1 \cdot \cos x}$$

$$\frac{1 - \cos^2 x}{\cos x}$$

$$\frac{\sin^2 x}{\cos x}$$

$$\frac{\sin x \cdot \boxed{\sin x}}{\cos x}$$

$$\frac{\sin x}{1} \cdot \frac{\sin x}{\cos x}$$

$$\frac{1}{\csc x} \cdot \frac{\tan x}{1}$$

$$\frac{\tan x}{\csc x} \checkmark$$

Sec 3.3 Sum and Difference for Cosine

Given

* $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$ *

$\cos(75^\circ)$

$\cos(\overset{\alpha}{30^\circ}+\overset{\beta}{45^\circ}) = \cos 30^\circ \cos 45^\circ - \sin 30 \sin 45$

$= \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} - \frac{1}{2}\frac{\sqrt{2}}{2}$

$= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$

$= \frac{\sqrt{6}-\sqrt{2}}{4}$

$15 = 45-30$

$45+{}^-30$

$\cos 90 = 0$

$105 = 60+45$

$135+{}^-30$

$\cos(60+30) = \cos 60 \cos 30 - \sin 60 \sin 30$

$= \frac{1}{2}\cdot\frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2}\cdot\frac{1}{2}$

$90 = 60+30$

$= \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4}$

$= 0$

$\cos(\alpha-\beta)$

$\cos(\alpha+{}^-\beta) = \cos(\alpha)\cos(-\beta) - \sin(\alpha)\sin(-\beta)$

$\cos(\alpha)\cos(\beta) - {}^-\sin(\alpha)\sin(\beta)$

$\boxed{\cos(\alpha-\beta) = \cos(\alpha)\cos(\beta) + \sin(\alpha)\sin(\beta)}$

Find:

$$\overset{\alpha}{\cos(61^\circ)}\,\overset{\beta}{\cos(16^\circ)} + \overset{\alpha}{\sin(61^\circ)}\,\overset{\beta}{\sin(16^\circ)}$$

$$\cos(61^\circ - 16^\circ)$$

$$\cos 45^\circ$$

$$\sqrt{2}/2$$

#57

$$\overset{\alpha}{\cos(10^\circ)}\,\overset{\beta}{\cos(20^\circ)} - \overset{\alpha}{\sin(10^\circ)}\,\overset{\beta}{\sin(20^\circ)} = \frac{\sqrt{3}}{2}$$

$$\cos(10 + 20)$$

$$\cos 30^\circ = \frac{\sqrt{3}}{2} \checkmark$$

What if

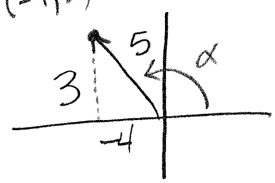
α is in Quad II with $\sin\alpha = 3/5$

β is in Quad I with $\sin\beta = 5/13$

Can I find $\cos(\alpha+\beta)$

α

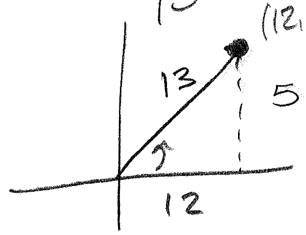
$(-4,3)$



$$\cos\alpha = \frac{-4}{5}$$

β

$(12,5)$



$$\cos\beta = \frac{12}{13}$$

$3^2 + x^2 = 5^2$

$9 + x^2 = 25$

$-9 \quad -9$

$x^2 = 16, \ x = 4$

$5^2 + x^2 = 13^2$

$25 + x^2 = 169$

$x^2 = 144, x = 12$

$$\cos(\alpha+\beta) =$$

$$\cos\alpha\cos\beta - \sin\alpha\sin\beta$$

$$\left(-\frac{4}{5}\right)\left(\frac{12}{13}\right) - \left(\frac{3}{5}\right)\left(\frac{5}{13}\right)$$

$$= \frac{-48 - 15}{65} = \boxed{-\frac{63}{65}}$$