

Trigonometry Identity and Formula Sheet

Sum Identities

$$\sin(\alpha + \beta) = \sin(\alpha) \cos(\beta) + \cos(\alpha) \sin(\beta)$$

$$\cos(\alpha + \beta) = \cos(\alpha) \cos(\beta) - \sin(\alpha) \sin(\beta)$$

$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$$

Product-to-Sum Identities

$$\sin A \cos B = \frac{1}{2} [\sin(A + B) + \sin(A - B)]$$

$$\sin A \sin B = \frac{1}{2} [\cos(A - B) - \cos(A + B)]$$

$$\cos A \sin B = \frac{1}{2} [\sin(A + B) - \sin(A - B)]$$

$$\cos A \cos B = \frac{1}{2} [\cos(A - B) + \cos(A + B)]$$

Reduction Formula

If α is an angle in standard position whose terminal side contains (a, b) and x is a real number then

$$a \sin x + b \cos x = \sqrt{a^2 + b^2} \sin(x + \alpha).$$

Area of a Triangle

$$A = \frac{1}{2} ab \cos \gamma$$

Angle Between Two Vectors

$$\cos \alpha = \frac{\vec{A} \cdot \vec{B}}{|\vec{A}| |\vec{B}|}$$

Roots of Complex Numbers

$$\sqrt[n]{z} = \sqrt[n]{r} \left[\cos\left(\frac{\theta + 2k\pi}{n}\right) + i \sin\left(\frac{\theta + 2k\pi}{n}\right) \right] \quad \text{for } k = 0, 1, 2, \dots, n - 1$$

Half-Angle Identities

$$\sin \frac{x}{2} = \pm \sqrt{\frac{1 - \cos x}{2}}$$

$$\cos \frac{x}{2} = \pm \sqrt{\frac{1 + \cos x}{2}}$$

$$\tan \frac{x}{2} = \pm \sqrt{\frac{1 - \cos x}{1 + \cos x}} = \frac{\sin x}{1 + \cos x} = \frac{1 - \cos x}{\sin x}$$

Sum-to-Product Identities

$$\sin x + \sin y = 2 \sin\left(\frac{x + y}{2}\right) \cos\left(\frac{x - y}{2}\right)$$

$$\sin x - \sin y = 2 \cos\left(\frac{x + y}{2}\right) \sin\left(\frac{x - y}{2}\right)$$

$$\cos x + \cos y = 2 \cos\left(\frac{x + y}{2}\right) \cos\left(\frac{x - y}{2}\right)$$

$$\cos x - \cos y = 2 \sin\left(\frac{x + y}{2}\right) \sin\left(\frac{x - y}{2}\right)$$

Heron's Formula

$$A = \sqrt{S(S - a)(S - b)(S - c)}$$

$$\text{where } S = \frac{a + b + c}{2}$$

Law of Cosines

$$c^2 = a^2 + b^2 - 2ab \cos \gamma$$

De Moivre's Theorem

$$z^n = r^n (\cos n\theta + i \sin n\theta)$$