6/6/2014 – Sec 1.5 (cont.), Sec 1.6, and Sec 2.1 (beginning)

Sec 1.3

#66 Want 14 oz

over/under 0.1

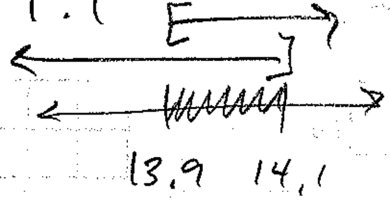
Weight = W

Equation: $|W-14| \leq 0.1$

$W-14 \leq 0.1$ <u>and</u> $W-14 \geq -0.1$

$+14 \quad +14 \qquad\qquad +14 \quad +14$

$W \leq 14.1 \qquad W \geq 13.9$



$[13.9, 14.1]$

$13.9 \leq W \leq 14.1$

Sec 1.4

\# 70

$$(A+B)^3 = A^3 + 3A^2B + 3AB^2 + B^3$$

$$(1-2i)^3 = (\underset{A}{1} + \underbrace{-2i}_{B})^3$$

$$= (1)^3 + 3(1)^2(-2i) + 3(1)(-2i)^2 + (-2i)^3$$

$$= 1 + 3 \cdot 1 \cdot -2i + 3 \cdot 1 \cdot 4\underset{(-1)}{i^2} + -8\underset{(-i)}{i^3}$$

$$= 1 + -6i + -12 + 8i$$

$$\boxed{-11 + 2i}$$

Sec 1.5 cont.

Completing the Square

Recall: $x^2 + 2ax + a^2 = (x+a)^2$

#59 $x^2 + 6x + \underline{\ 9\ } = (x+3)^2$

How to (Require coefficient of x^2 to be 1)

Recall $ax^2 + bx + c$ is standard form.

$a = 1$

1. take b and divide by 2 $\left(\frac{b}{2}\right)$
2. Square $\left(\frac{b}{2}\right)$
3. Add both sides of equation to keep Equation balanced.
4. Factor our Newly created Perfect Square trinomial

\# 69 $p^2 + 6p = -4$

$\frac{6}{2} = 3$

$(3)^2 = 9$

$p^2 + 6p + 9 = -4 + 9$

$\sqrt{(p+3)^2} = \pm\sqrt{5}$

$p + 3 = \pm\sqrt{5}$

$\quad -3 \quad -3$

$\boxed{p = -3 \pm \sqrt{5}}$

\# 74 $w^2 - 7w + 3 = 0$

$\qquad\qquad -3 \quad -3$

$-\frac{7}{2}$

$\left(-\frac{7}{2}\right)^2 = \frac{49}{4}$

$w^2 - 7w + \frac{49}{4} = -3 + \frac{49}{4}$

$\sqrt{\left(w - \frac{7}{2}\right)^2} = \pm\sqrt{\frac{37}{4}}$

$w - \frac{7}{2} = \pm\sqrt{\frac{37}{4}}$

$w - \frac{7}{2} = \frac{\pm\sqrt{37}}{\sqrt{4}}$

#74 cont.

$$w - \frac{7}{2} = \pm\frac{\sqrt{37}}{2}$$

$$+\tfrac{7}{2} \quad +\tfrac{7}{2}$$

$$w = \frac{7}{2} \pm \frac{\sqrt{37}}{2}$$

$$= \frac{7 \pm \sqrt{37}}{2}$$

Quadratic Formula

if $ax^2 + bx + c = 0$

then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

1st Get in standard form

2nd identify a, b, c

3rd Plug into Formula

4th Simplify

\# 103 $2a^2 + 5 = 3a$

$$\begin{aligned} 2a^2 + 5 &= 3a \\ -3a \quad &\quad -3a \end{aligned}$$

$$2a^2 - 3a + 5 = 0$$

$a = 2 \quad b = -3 \quad c = 5$

$$a = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(5)}}{2(2)}$$

$$= \frac{3 \pm \sqrt{9 - 40}}{4}$$

$$= \frac{3 \pm \sqrt{-31}}{4}$$

$$\boxed{a = \frac{3 \pm i\sqrt{31}}{4}}$$

Solution types: Rational Numbers,
irrational Numbers,
Complex Numbers
Repeated Root

Ex: $x^2 - 6x + 9 = 0$

$$\sqrt{(x-3)^2} = \pm\sqrt{0}$$

$$x - 3 = \pm 0$$

$$+3 \quad +3$$

$$x = 3.$$

The discriminant tells us what type of solutions we get from our Quadratic

Let $ax^2 + bx + c = 0$

Discriminant $= b^2 - 4ac$

if Discriminant

= 0 then Repeated Roots

is (+) and perfect Square then Rational

is (+) and Not a perfect Square then irrational

is (−) then Complex

Sec 1.6 Solving Othertypes of Equations.

Polynomials of Higher Degree

If the polynomial can be factored, the zero product property can be used.

#11 $2x^4 - 3x^3 = 9x^2$

$\quad -9x^2 \quad -9x^2$

$$2x^4 - 3x^3 - 9x^2 = 0$$

$$x^2(2x^2 - 3x - 9) = 0$$

$$x^2(x-3)(2x+3) = 0$$

-18	-3
-6, 3	-3 ✓

$\sqrt{x^2} = \sqrt{0}$ $\quad x - 3 = 0$ $\quad 2x + 3 = 0$ $\quad \frac{-6}{2} = -3.$

$\quad +3 \; +3 \quad -3 \;\; -3$

$x = \pm 0$ $\quad \frac{2x}{2} = \frac{-3}{2}$ $\quad \frac{3}{2} = \frac{3}{2}.$

$x = 0$ $\quad x = 3$ $\quad x = -3/2$

$$\{0, -3/2, 3\}$$

Rational Equations

Goal: Clear the fractions using the LCD.

Note: Be sure any answer will not make the original Denominator equal to zero

#41 $\dfrac{6}{n+3} + \dfrac{20}{n^2+n-6} = \dfrac{5}{n-2}$

LCD: $(n+3)(n-2)$

D: $n \neq -3, 2$

$(n+3)(n-2)\left[\dfrac{6}{n+3} + \dfrac{20}{(n+3)(n-2)}\right] = \left[\dfrac{5}{n-2}\right](n+3)(n-2)$

$\dfrac{6\cancel{(n+3)}(n-2)}{\cancel{n+3}} + \dfrac{20\cancel{(n+3)}\cancel{(n-2)}}{\cancel{(n+3)}\cancel{(n-2)}} = \dfrac{5(n+3)\cancel{(n-2)}}{\cancel{n-2}}$

$6(n-2) + 20 = 5(n+3)$

$6n - 12 + 20 = 5n + 15$

$6n + \cancel{8} = \cancel{5n} + 15$

$-5n \quad -\cancel{8} \quad -\cancel{5n} \quad -8$

$\boxed{n = 7}$

Radical equations and Equations with Rational Exponents

Use the Power Property of Equality

If $\sqrt[n]{u} = v$ then $\left(\sqrt[n]{u}\right)^n = v^n$

$u = v^n$ for $n \geq 2$

Note: Check Every Answer in <u>original</u> equation

#57

$x = 3 + \sqrt{23 - x}$

$-3 \quad -3$

$(x-3)^2 = \left(\sqrt{23 - x}\right)^2$

Work
$(x-3)(x-3)$
$x^2 - 3x - 3x + 9$

$x^2 - 6x + 9 = 23 - x$

$+x \quad -23 \quad -23 \quad +x$

$x^2 - 5x - 14 = 0$

-14	-5
-7, 2	-5

$(x-7)(x+2) = 0$

$x - 7 = 0 \quad x + 2 = 0$

$x = 7 \quad x = -2$

1st Isolate the Radical

2nd Apply the power property

3rd Solve the Resulting polynomial

4th Check in original Equation

Check

$\cancel{x = -2}$

$x = 3 + \sqrt{23 - x}$

$-2 = 3 + \sqrt{23 + 2}$

$-2 = 3 + \sqrt{25}$

$-2 = 3 + 5$

$-2 \neq 8$

Check

$x = 7$

$7 = 3 + \sqrt{23 - 7}$

$7 = 3 + \sqrt{16}$

$7 = 3 + 4$

$7 = 7$ ✓

$\boxed{\text{Ans: } \{7\}}$

When dealing with Rational Exponents Remember they are exponents and Radicals

$$x^{3/4} = \sqrt[4]{x^3}$$

$$\left(x^{3/4}\right)^{4/3} = 2^{4/3}$$

$$x = 2^{4/3}$$

Equations Quadratic in Form

Use a u-substitution to change to a quadratic.

#69 $(x^2-3)^2 + (x^2-3) - 2 = 0$

$u = x^2-3$

$u^2 + u - 2 = 0$

$(u+2)(u-1) = 0$

$u+2=0 \quad , \quad u-1=0$

$-2\ -2 \qquad\quad +1\ \ +1$

$u = -2 \qquad u = 1$

$x^2-3 = -2 \qquad x^2-3 = 1$

$+3\ \ +3 \qquad\quad +3\ \ +3$

$\sqrt{x^2} = \pm\sqrt{1} \qquad \sqrt{x^2} = \pm\sqrt{4}$

$x = \pm 1 \qquad x = \pm 2$

$\{-1, 1, -2, 2\}$

Work Problem

$$\frac{1}{\text{time alone}}\left(\text{time together}\right) + \frac{1}{\text{time alone}}\left(\frac{\text{time}}{\text{together}}\right) = 1$$

Distance - Rate - time

	Rate *	time =	distance
with wind	$X+5$		
against wind	$X-5$		

$rt = d$

$t = \frac{d}{r}$

$r = \frac{d}{t}$

car speed $= X$

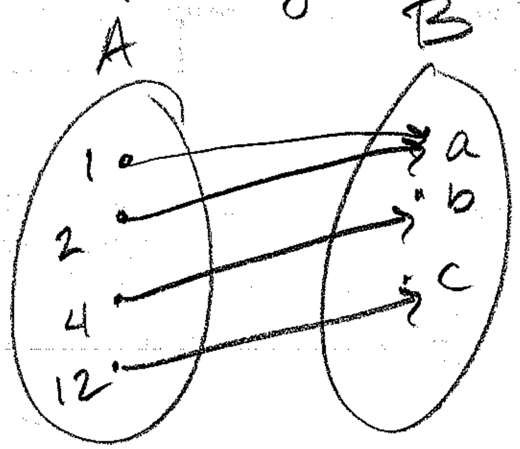
wind speed $= 5$

Sec 2.1 Rectangular Coordinates Graphing Circles and Other Relations

A **Relation** is a correspondence between two sets

We can express or Represent a Relation in a couple ways

① mapping



Notation $A \to B$

② ordered pairs

$(1, a)$

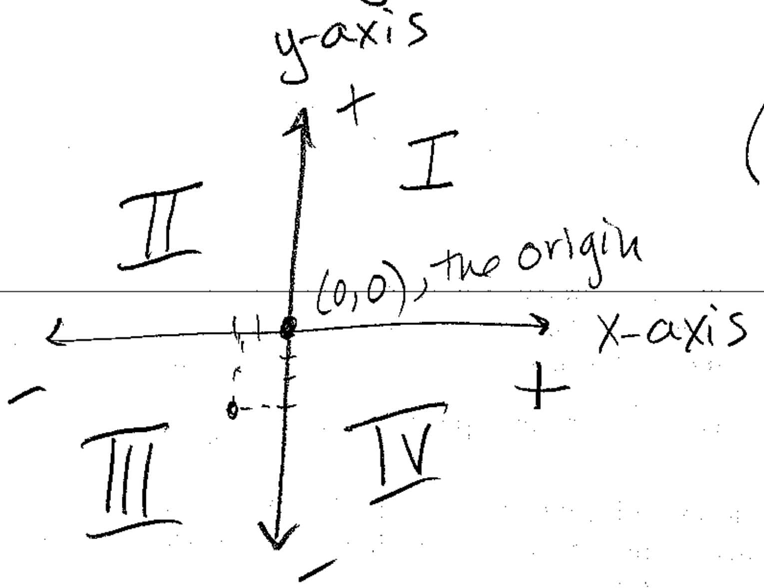
independent Variable (inputs)

dependent Variable (outputs)

the set of inputs = Domain

the set of outputs = Range

Rectangular Coordinate System



(x-coordinate, y-coordinate)

Graph $(-2, -3)$

#16 $|y+1| = x$

x	y
0	−1
1	0, −2
3	2, −4
5	4, −6
6	5, −7
7	6, −8

$|y+1| = 0$

$y+1=0$ or $y+1=-0$

$\underline{-1\ \ -1}$

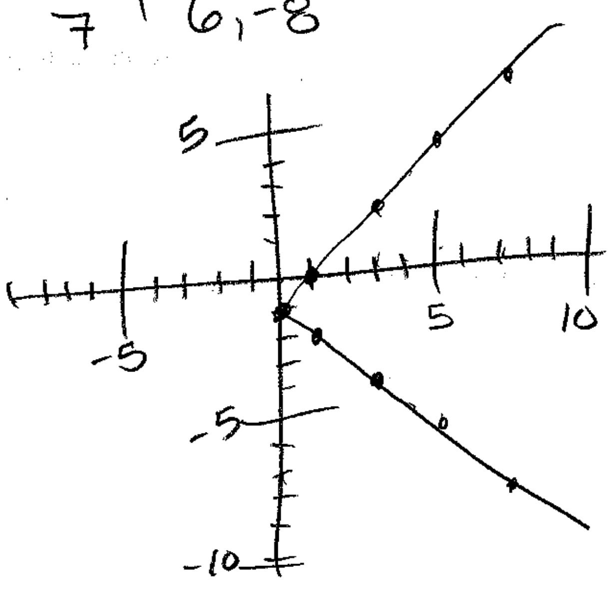
$y = -1$

$|y+1| = 1$

$y+1=1$ or $y+1=-1$

$\underline{-1\ \ -1}$ $\qquad$ $-1\ \ -1$

$y=0$ $\qquad$ $y=-2$

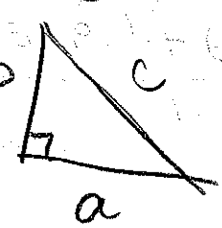


Given 2 points, (x_1, y_1) (x_2, y_2)

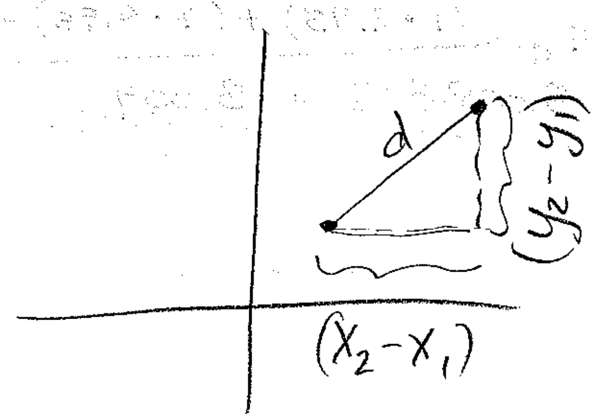
Midpoint: $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$

Distance: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Recall Pythagorean's Theorem
Given a Right triangle

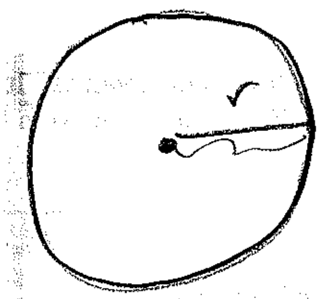


$a^2 + b^2 = c^2$



Circles

Defn: A circle can be defined as the set of all points in a plane that are a fixed distance (called the Radius) from a fixed point (called the center)



Standard Form of a Circle if (h,k) is the center and r is the Radius, then

$$(x-h)^2 + (y-k)^2 = r^2$$

is the standard Form

Circle with center at $(-2,3)$
with $R = 5$

1. Find Equation
2. Graph

Equation

$$(x - {}^{-}2)^2 + (y-3)^2 = 5^2$$

$$(x+2)^2 + (y-3)^2 = 25$$

center $(-2,3)$
Radius: 5