6/3/2014 - Sec R.5

R.4

#56

$$V = \frac{4}{3}\pi R^3 - \frac{4}{3}\pi r^3$$

$$\frac{4}{3}\pi\left(R^3 - r^3\right)$$

$a^3 - b^3$

$(a-b)(a^2+ab+b^2)$

$$\frac{4}{3}\pi(R-r)(R^2+Rr+r^2)$$

$V = ?$ $R = 1.8$, $r = 1.5$

$$V = \frac{4}{3}\pi(1.8-1.5)(1.8^2 + 1.8(1.5) + 1.5^2)$$

$$\frac{4}{3}\pi(.3)(3.24 + 2.7 + 2.25)$$

$$\frac{4}{3}\pi(.3)(8.19)$$

$$V = 10.2919 \text{ cm}^3$$

R.4

#63

$$192x^3 - 164x^2 - 270x$$

$$2x(96x^2 - 82x - 135)$$

$$2x(16x - 27)(6x + 5)$$

-12960	-82
-162.80	-82 ✓

$$\frac{-162}{96} = \frac{-27}{16}$$

$$\frac{80}{96} = \frac{5}{6}$$

#68 $m^6 - 64$ (even) difference of squares

$$(m^3)^2 - 8^2$$

$$(m^3 - 8)(m^3 + 8)$$

(m^3) $(2)^3$ (m^3) $(2)^3$

$$(m-2)(m^2+2m+2^2)(m+2)(m^2-2m+2^2)$$

$$(m-2)(m^2+2m+4)(m+2)(m^2-2m+4)$$

$$(m^3)^2 = m^6$$

$$(m \cdot m \cdot m \cdot m \cdot m \cdot m)$$

6 m's

$$(m^3) = m^3 \cdot m^3$$

R.5 Rational Expressions

Defn: a Rational Expression is one that can be written as the Quotient of two polynomials.

It is in Simplest Form when the Numerator and denominator have NO common factors

#8a $\dfrac{x-4}{-7x+28}$ ←

$$\frac{\cancel{x-4}}{-7(\cancel{x-4})} = \frac{1}{-7} = -\frac{1}{7} = \boxed{\frac{-1}{7}}$$

#10b $\dfrac{m^2+3m-4}{m^2-4m}$ ← Ephraim ← Richfield

$$\frac{(m+4)(m-1)}{m(m-4)}$$ Simplest Form

Things to Watch for

$$\frac{m}{m+4} = \frac{1}{4}$$

$$\frac{m}{(m+4)} = \frac{m}{m+4}$$

(m+4): term, factor

Recall Multiplication: $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$

Ex #19 $\frac{x^2-7x-18}{x^2-6x-27} \cdot \frac{2x^2+7x+3}{2x^2+5x+2}$

1st multiply together

$$\frac{\text{Back Row}\ (x^2-7x-18)(2x^2+7x+3)\ \text{front Row}}{\text{Middle Row}\ (x^2-6x-27)(2x^2+5x+2)\ \text{Richfield}}$$

2nd factor everything

$$\frac{(x+2)(x-9)(2x+1)(x+3)}{(x-9)(x+3)(x+2)(2x+1)}$$

$$\boxed{1}$$

Division

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$$

flip and multiply

#21 $\dfrac{p^3-64}{p^3-p^2} \div \dfrac{p^2+4p+16}{p^2-5p+4}$

1st row / 3rd row: $\dfrac{p^3-64}{p^3-p^2}$ · 2nd row: $\dfrac{p^2-5p+4}{p^2+4p+16}$ Richfield

$$\frac{(p-4)(p^2+4p+16)(p-1)(p-4)}{p^2(p-1)(p^2+4p+16)}$$

$$\frac{(p-4)^2}{p^2}$$ ✓

Adding / Subtracting Fractions:

Need Common Denominators (aka LCD)

How to:

1. Factor Both Denominators to find LCD
2. Build Equivalent Expressions using the LCD
3. Add / Subtract Numerator as indicated
4. Reduce Result to Lowest Terms.

#58 $\dfrac{-2}{3a+12} - \dfrac{7}{a^2+4a}$

$\dfrac{-2}{3(a+4)} - \dfrac{7}{a(a+4)}$ LCD: $3a(a+4)$

$\dfrac{-2a}{3(a+4)\cdot a} - \dfrac{7\cdot 3}{a(a+4)\cdot 3}$

$$\frac{-2a-21}{3a(a+4)}$$

Simplifying Complex Fractions

Ex of Complex Fraction

$$\frac{\frac{2}{3m} - \frac{3}{2}}{\frac{3}{4m} - \frac{1}{3m^2}}$$

2 methods to Simplify

Method 1

1. get a single fraction numerator and single fraction denominator
2. flip the denominator fraction and multiply.
3. Simplify to lowest terms

LCD $6m$

$$\frac{\frac{2\cdot 2}{3m\cdot 2}-\frac{3\cdot 3m}{2\cdot 3m}}{\frac{3m\cdot 3}{3m\cdot 4m}-\frac{1\cdot 4}{3m^2\cdot 4}}=\frac{\frac{4-9m}{6m}}{\frac{9m-4}{12m^2}}$$

LCD $12m^2$

$$\frac{-1(-4+9m)}{(4-9m)}\cdot\frac{2m}{(9m-4)}$$

$$\frac{-1(9m-4)}{1}\cdot\frac{2m}{(9m-4)}$$

$$-1\cdot 2m$$

$$\boxed{-2m}$$

#67

$$\frac{\frac{p}{1}+\frac{1}{p-2}}{\frac{1}{1}+\frac{1}{p-2}}$$

Numerator
LCD: $p-2$

Denominator
LCD: $p-2$

$$\frac{\frac{p(p-2)}{p-2}+\frac{1}{p-2}}{\frac{1(p-2)}{p-2}+\frac{1}{p-2}}$$

$$\frac{\frac{p^2-2p+1}{p-2}}{\frac{p-2+1}{p-2}}$$

$$\frac{\frac{p^2-2p+1}{p-2}}{\frac{p-1}{p-2}} \quad \text{flip} \quad = \frac{p^2-2p+1}{(p-2)} \cdot \frac{(p-2)}{(p-1)}$$

$$\frac{(p-1)(p-1)\cancel{(p-2)}}{\cancel{(p-2)}\cancel{(p-1)}}$$

$$\boxed{p-1}$$

Method 2

1. Find LCD for every fraction in the Numerator and Denominator
2. Multiply the Numerator and Denominator to clear the double fraction
3. Simplify to Lowest terms

$$\frac{\left(\frac{2}{3m} - \frac{3}{2}\right)12m^2}{\left(\frac{3}{4m} - \frac{1}{3m^2}\right)12m^2}$$

everything LCD: $12m^2$

$4 \cdot 3 \cdot 3m^2$

$$\frac{2 \cdot \frac{4}{12}\cancel{m}\cancel{m} - 3 \cdot \frac{6}{12}m^2}{3 \cdot \frac{3}{12}\cancel{m}\cancel{m}^2 - 1 \cdot \frac{4}{12}\cancel{m}\cancel{m}^2} = \frac{8m - 18m^2}{9m - 4}$$

$$\frac{-2m(-4 + 9m)}{9m - 4}$$

$$\frac{-2m}{1} = \frac{-2m(9m - 4)}{(9m - 4)}$$

#67 again

$$\frac{\left(p + \frac{1}{p-2}\right)(p-2)}{\left(1 + \frac{1}{p-2}\right)(p-2)} \qquad \text{LCD: } (p-2)$$

$$\frac{p(p-2) + \frac{1\cdot(p-2)}{p-2}}{1\cdot(p-2) + \frac{1\cdot(p-2)}{p-2}}$$

$$\frac{p(p-2) + 1}{1\cdot(p-2) + 1}$$

$$\frac{p^2 - 2p + 1}{p - 2 + 1}$$

$$\frac{p^2 - 2p + 1}{p - 1}$$

$$\frac{(p-1)(p-1)}{p-1} = \boxed{p-1}$$