Homework Questions 10.2 #15, 36(a)(b), 24

(15) $6^x = 36$

$6^x = 6^2$

$x = 2$

$\{2\}$

(24) $10^x = 0.1$

$10^x = \frac{1}{10}$

$10^x = 10^{-1}$

$x = -1$

$\{-1\}$

$10^{-1} = \frac{1}{10} = 0.1$

(36) a) $y = (1.046 \times 10^{-38})(1.0444^x)$

year 2040

$y = (1.046 \times 10^{-38})(\underline{1.0444}^{\underline{2040}})$

$y^x \quad \wedge \quad x^{\square}$

$= (1.046 \times 10^{-38})(3.079 \times 10^{38})$

$= 3.220634 \times 10^0$

$\boxed{3.2°C}$

b) $y = 0.009x - 17.67$

$= 0.009(2040) - 17.67$

$= 0.69\ °C$

$\doteq \boxed{0.7°C}$

10.3 Logarithmic Functions

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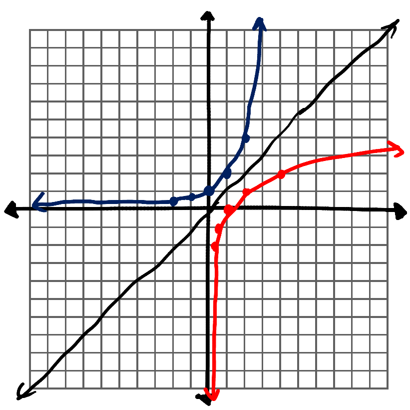
Objectives:

- Define a logarithm.
- Convert between exponential and logarithmic forms.
- Solve logarithmic equations of the form $\log_a b = k$ for a, b, or k.
- Define and graph logarithmic functions.
- Use logarithmic functions in applications involving growth or decay.

Review

- Graph $y = 2^x$

x	y
-2	$\frac{1}{4}$
-1	$\frac{1}{2}$
0	1
1	2
2	4



- Find and graph it's inverse.

$x = 2^y$

x	y
$\frac{1}{4}$	-2
$\frac{1}{2}$	-1
1	0
2	1
4	2

$\boxed{x = 2^y}$

means the same:

$\boxed{y = \log_2 x}$

$\sqrt{2} = 2^{\frac{1}{2}}$

Logarithm

For all positive numbers a, with $a \neq 1$, and all positive numbers x

$\underbrace{y}_{\text{exp.}} = \log_{\underbrace{a}_{\text{base}}} x$ means the same as $x = \underbrace{a}_{\text{base}}^{y \to \text{exp.}}$

read it = "y equals the log base a of x"

Meaning of $\log_a x$

A logarithm is an exponents.

The expression $\log_a x$ represents the exponent to which the base a must be raised to obtain x.

Converting Between Exponential and Logarithmic Forms

- Fill in the blanks with the equivalent forms.

Exponential Form	Logarithmic Form
$3^2 = 9$	$\log_3 9 = 2$
$\left(\frac{1}{5}\right)^{-2} = 25$	$\log_{\frac{1}{5}} 25 = -2$
$10^5 = 100{,}000$	$\log_{10} 100{,}000 = 5$
$4^{-3} = \frac{1}{64}$	$\log_4 \frac{1}{64} = -3$

- Fill in the blanks with the equivalent forms.

Exponential Form	Logarithmic Form
$2^5 = 32$	$\log_2 32 = 5$
$100^{\frac{1}{2}} = 10$	$\log_{100} 10 = \frac{1}{2}$
$8^{\frac{2}{3}} = 4$	$\log_8 4 = \frac{2}{3}$
$6^{-4} = \frac{1}{1296}$	$\log_6 \frac{1}{1296} = -4$
$\sqrt[3]{64} = 4$	$\log_{64} 4 = \frac{1}{3}$

$64^{\frac{1}{3}} = 4$

Solving Logarithmic Equations

equation w/ logs

Solve each equation.

$\log_4 x = -2$

rewrite in exp. form

$4^{-2} = x$

$\frac{1}{4^2} = x$

$\frac{1}{16} = x$

$\left\{\frac{1}{16}\right\}$

$\log_{\frac{1}{2}}(3x + 1) = 2$

$\left(\frac{1}{2}\right)^2 = 3x + 1$

$\frac{1}{4} = 3x + 1$

$-\frac{4}{4} \quad -1$

$\frac{-\frac{3}{4}}{3} = \frac{\cancel{3}x}{\cancel{3}}$

$\boxed{\left\{-\frac{1}{4}\right\}}$ $-\frac{3}{4} \div \frac{3}{1} = -\frac{\cancel{3}}{4} \cdot \frac{1}{\cancel{3}} = \boxed{-\frac{1}{4}}$

Solving Logarithmic Equations

Solve each equation.

$\log_x \mathbf{3} = \mathbf{2}$

$\sqrt{x^2} = \pm\sqrt{3}$

$x = \pm\sqrt{3}$

$\sqrt{3},\ \cancel{-\sqrt{3}}$

base can't be negative

$\log_{\sqrt{3}} 3 = 2$

$\sqrt{3}^2 = 3$

$\boxed{\{\sqrt{3}\}}$

$\log_{49} \sqrt[3]{7} = x$

$49^x = \sqrt[3]{7}$

$(7^2)^x = (7)^{\frac{1}{3}}$

$7^{2x} = 7^{\frac{1}{3}}$

$\frac{1}{2} \cdot 2x = \frac{1}{3} \cdot \frac{1}{2}$

$x = \frac{1}{6}$

$\boxed{\{\frac{1}{6}\}}$

Properties of Logarithms

For any positive real number b, with $b \neq 1$, the following are true.

$\log_b b = 1$ (because $b^1 = b$) and $\log_b 1 = 0$ (because $b^0 = 1$)

- Evaluate each logarithm.

$\log_7 \mathbf{7} = 1$

$\log_{\sqrt{2}} \sqrt{2} = 1$

$\log_9 \mathbf{1} = 0$

$\log_{0.2} 1 = 0$

Logarithmic Function

If a and x are positive numbers, with $a \neq 1$, then

$$g(x) = \log_a x$$

defines the **logarithmic function with base a.**

- Graph $g(x) = \log_3 x$

$y = \log_3 x$

No y-int

$\boxed{3^y = x}$

x-int (1,0)

x	y
$\frac{1}{9}$	-2
$\frac{1}{3}$	-1
1	0
3	1
9	2

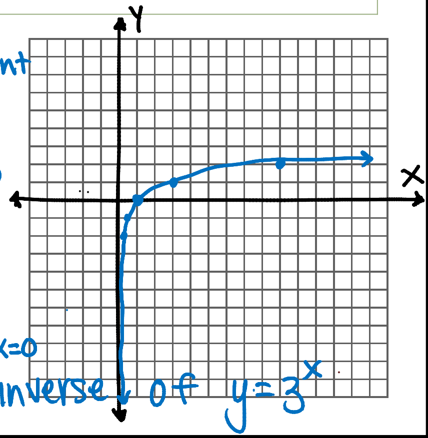
Increasing

Domain: $(0,\infty)$

Range: $(-\infty,\infty)$

VA: y-axis AKA: x=0

Function is the inverse of $y=3^x$



Graphing a Logarithmic Function

- Graph $f(x) = \log_{\frac{1}{2}} x$

$\left(\frac{1}{2}\right)^y = x$

x	y
1	☺
½	1
¼	2
2	-1
4	-2
8	-3

Decreasing

x-int: (1,0)

y-int: none

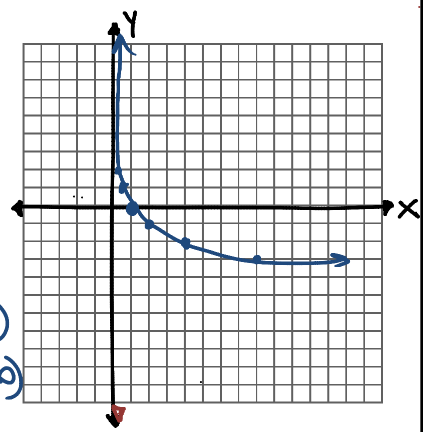
Domain: $(0,\infty)$

Range: $(-\infty,\infty)$

VA: x=0

- Function is inverse of $y=\left(\frac{1}{2}\right)^x$

$f(x)=\left(\frac{1}{2}\right)^x$



Characteristics of the Graph of $g(x) = \log_a x$

Logarithmic Function

- The graph contains the point (1,0).
- The function is one-to-one. Increasing
    - When $a > 1$, the graph will rise from left to right, from the fourth quadrant to the first.
    - When $0 < a < 1$, the graph will fall from left to right, from the first quadrant to the fourth. Decreasing
- The graph will approach the y-axis, but never touch it. (The y-axis is an asymptote.)
- The domain is $(0,\infty)$, and the range is $(-\infty,\infty)$.

Solving an Application of a Logarithmic Function

- Suppose the gross national product (GNP) of a small country (in millions of dollars) is approximated by

$$G(t) = 15.0 + 2.00 \log_{10} t$$

Where t is the time in years since 2003.

- Approximate to the nearest tenth the GNP for $\boxed{t = 1}$ and $\boxed{t = 10}$.

$G(1) = 15.0 + 2.00\boxed{\log_{10} 1}$

$G(1) = 15.0 + \cancel{2.00(0)}$

$\boxed{G(1) = \$15 \text{ million}}$

$G(10) = 15.0 + 2.00\boxed{\log_{10} 10}$ =1

$G(10) = 15.0 + 2.00(1)$

$\boxed{G(10) = \$17 \text{ million}}$