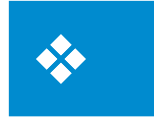


Appendix H
The Binary Number System and Bitwise Operations

One of the many powers that C++ gives the programmer is the ability to work with the individual bits of an integer field. The purpose of this appendix is to give an overview of how integer types are stored in binary and explain the bitwise operators that the C++ offers. Finally, we will look at bit fields, which allow us to treat the individual bits of a variable as separate entities.

Integer Forms

The integer types that C++ offers are as follows:

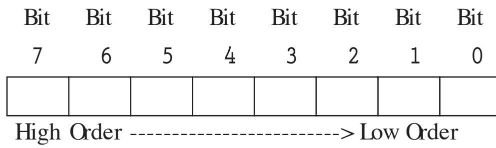
```
char
int
short
long
unsigned char
unsigned (same as unsigned int)
unsigned short
unsigned long
```

When you assign constant values to integers in C++, you may use decimal, octal, or hexadecimal. Placing a zero in the first digit creates an octal constant. For example, 0377 would be interpreted as an octal number. Hexadecimal constants are created by placing 0x or 0X (zero-x, not O-x) in front of the number. The number 0X4B would be interpreted as a hex number.

Binary Representation

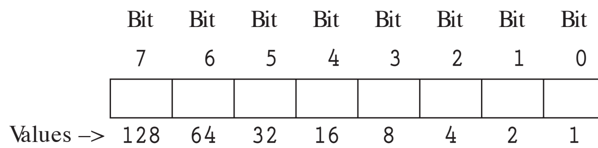
Regardless of how you express constants, integer values are all stored the same way internally—in binary format. Let's take a quick review of binary number representation.

Let's assume we have a one-byte field. The diagram below shows our field broken into its individual bits.



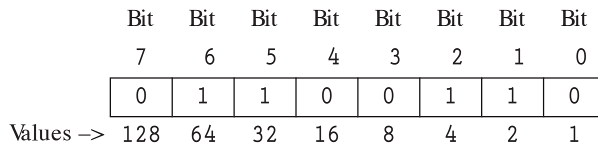
The leftmost bits are called the *high order* bits and the rightmost bits are called the *low order* bits. Bit number 7 is the highest order bit, so it is called the *most significant* bit.

Each of these bits has a value assigned to it. The diagram below shows the values of each bit:



These values are actually powers of two. The value of bit 0 is 2^0, which is 1. The value of bit 1 is 2^1, which is 2. Bit 2 has the value 2^2, which is 4. It progresses to the last bit.

When a number is stored in this field, the bits may be set to either 1 or 0. Here is an example.



Here, bits 1, 2, 5, and 6 are set to 1. To calculate the overall value of this bit pattern, we add up all of the bit values of the bits that are set to 1.

Bit 1's value	2
Bit 2's value	4
Bit 5's value	32
Bit 6's value	64
	—
Overall Value	102

The bit pattern `01100110` has the decimal value 102.

Negative Integer Values

One way that a computer can store a negative integer is to use the leftmost bit as a sign bit. When this bit is set to 1, it would indicate a negative number, and when it is set to 0 the number would be positive. The problem with this, however, is that we would have two bit patterns for the number 0. One pattern would be for positive 0, the other would be for negative 0. Because of this, most systems use two's complement representation for negative integers.

To calculate the two's compliment of a number, first you must get the one's compliment. This means changing each 1 to a 0, and each 0 to a 1. Next, add 1 to the resulting number. What you have is the two's compliment. Here is how the computer stores the value –2.

2 is stored as	00000010
Get the one's compliment	11111101
Add 1	1
	____
And the result is	11111110

As you can see, the highest order bit is still set to 1, indicating not only that this is a negative number, but it is stored in two's compliment representation.

Bitwise Operators

C++ provides operators that let you perform logical operations on the individual bits of integer values, and shift the bits right or left.

The Bitwise Negation Operator

The bitwise negation operator is the ~ symbol. It is a unary operator that performs a negation, or one's compliment on each bit in the field. The expression

```
~val
```

returns the one's compliment of `val`. It does not change contents of `val`. It could be used in the following manner:

```
negval = ~val;
```

This will store the one's compliment of `val` in `negval`.

The Bitwise AND Operator

The bitwise AND operator is the & symbol. This operator performs a logical AND operation on each bit of two operands. This means that it compares the two operands bit by bit. For each position, if both bits are 1, the result will be 1. If either or both bits are 0, the results will be 0. Here is an example:

```
andval = val & 0377;
```

The result of the AND operation will be stored in `andval`. There is a combined assignment version of this operator. Here is an example:

```
val &= 0377;
```

This is the same as

```
val = val & 0377;
```

The Bitwise OR Operator

The bitwise OR operator is the | symbol. This operator performs a logical OR operation on each bit of two operands. This means that it compares the two operands bit by bit. For each position, if either of the two bits is 1, the result will be 1. Otherwise, the results will be 0. Here is an example:

```
orval = val | 0377;
```

The result of the OR operation will be stored in `orval`. There is a combined assignment version of this operator. Here is an example:

```
val |= 0377;
```

This is the same as

```
val = val | 0377;
```

The Bitwise EXCLUSIVE OR Operator

The bitwise EXCLUSIVE OR operator is the ^ symbol. This operator performs a logical XOR operation on each bit of two operands. This means that it compares the two operands bit by bit. For each position, if one of the two bits is 1, but not both, the result will be 1. Otherwise, the results will be 0. Here is an example:

```
xorval = val ^ 0377;
```

The result of the XOR operation will be stored in `xorval`. There is a combined assignment version of this operator. Here is an example:

```
val ^= 0377;
```

This is the same as

```
val = val ^ 0377;
```

Using Bitwise Operators with Masks

Suppose we have the following variable declarations:

```
char value = 110, cloak = 2;
```

The binary pattern for each of these two variables will be as follows:

value -->	0	1	1	0	1	1	1	0	= 110
cloak -->	0	0	0	0	0	0	1	0	= 2

The operation

```
value &= cloak;
```

will perform a logical bitwise AND on the two variables `value` and `cloak`. The result will be stored in `value`. Remember a bitwise AND will produce a 1 only when both bits are set to 1. Here is a diagram showing the result of the AND operation.

value -->	0	1	1	0	1	1	1	0

AND

cloak -->	0	0	0	0	0	0	1	0

equals

result -->	0	0	0	0	0	0	1	0

The 0's in the `cloak` variable "hide" the values that are in the corresponding positions of the `value` variable. This is called *masking*. When you mask a variable, the only bits that will "show through" are the ones that correspond with the 1's in the mask. All others will be turned off.

Turning Bits On

Sometimes you may want to turn on selected bits in a variable and leave all of the rest alone. This operation can be performed with the bitwise OR operator. Let's see what happens when we OR the `value` and `cloak` variables we used before, but using a different value for cloak.

```
char value = 110, cloak = 16;
value |= mask;
```

This diagram illustrates the result of the OR operation:

value -->	0	1	1	0	1	1	1	0

OR

cloak -->	0	0	0	1	0	0	0	0

equals

result -->	0	1	1	1	1	1	1	0

This caused bit 4 to be turned on and all the rest to be left alone.

Turning Bits Off

Suppose that instead of turning specific bits on, you wish to turn them off. Assume we have the following declaration:

```
char status = 127, cloak = 8;
```

Bit 3 of `cloak` is set to 1, and all the rest are set to 0. If we wish to set bit 3 of status to 0 we must AND it with the negation of `cloak`. In other words, we must get the one's compliment of `cloak`, then AND it with `status`. The statement would look like this:

```
status &= ~cloak;
```

Here is what `cloak`'s bit pattern looks like:

cloak -->	0	0	0	0	1	0	0	0

And here is the one's compliment of `cloak`:

~cloak -->	1	1	1	1	0	1	1	1

Here is what we get when we AND status and the one's compliment of `cloak`:

status -->	0	1	1	1	1	1	1	1

AND

~cloak -->	1	1	1	1	0	1	1	1

equals

result -->	0	1	1	1	0	1	1	1

Bit 3 of status is turned off, and all other bits were left unchanged.

Toggling Bits

To toggle a bit is to flip it off when it is on, and on when it is off. This can be done with the EXCLUSIVE OR operator. We will use the following variables to illustrate.

```
char status = 127, cloak = 8;
```

Our objective is to toggle bit 3 of `status`, so we will use the XOR operator.

```
status ^= cloak;
```

Here is the diagram of the operation:

status -->	0	1	1	1	1	1	1	1

XOR

cloak -->	0	0	0	0	1	0	0	0

equals

result -->	0	1	1	1	0	1	1	1

Bit 3 of `status` will be set to 0. If we repeat this operation with the new value of `status`, this is what will happen:

status -->	0	1	1	1	0	1	1	1

XOR

cloak -->	0	0	0	0	1	0	0	0

equals

result -->	0	1	1	1	1	1	1	1

Bit 3 of `status` will be toggled again, restoring it to its previous state.

Testing the Value of a Bit

To test the value of an individual bit, you must use the AND operator. For example, if we want to test the variable `bitvar` to see if bit 2 is on, we must use a mask that has bit 2 turned on. Here is an example of the test:

```
if ((bitvar & 4) == 4)
    cout << "Bit 2 is on.\n";
```

Remember that ANDing a value with a mask will produce a value that hides all of the bits but the ones turned on in the mask. If bit 2 of `bitvar` is on, the expression `bitvar & 4` will return the value 4.

The parentheses around `bitvar & 4` are necessary because the == operator has higher precedence than the `&` operator.

The Bitwise Left Shift Operator

The bitwise left shift operator is two less-than signs (`<<`). It takes two operands. The operand on the left is the one to be shifted, and the operand on the right is the number of places to shift. When the bit values are shifted left, the vacated positions to the right are filled with 0 and the bits that shift out of the field are lost. Suppose we have the following variables:

```
char sand = 2, shiftsand;
```

The following statement will store in `shiftsand` the value of `sand` shifted left two places.

```
shiftsand = sand << 2;
```

Let's see what is happening behind the scenes with the value in `sands`

Before shift	0	0	0	0	0	0	1	0
After shift	0	0	0	0	1	0	0	0

Realize, however that `sand` itself is not being shifted. The variable `shiftsand` is being set to the value of `sand` shifted left two places. If we wanted to shift `sand` itself, we could use the combined assignment version of the left shift operator.

```
sand <<= 2;
```

Shifting a number left by n places is the same as multiplying it by 2^n. So the example above is the same as

```
sand *= 4;
```

The bitwise shift will almost always work faster, however.

The Bitwise Right Shift Operator

The bitwise right shift operator is two greater-than signs (>>). Like the left shift operator, it takes two operands. The operand on the left is the one to be shifted, and the operand on the right is the number of places to shift. When the bit values are shifted right, and the variable is signed, what the vacated positions to the left are filled with depends on the machine. They could be filled with 0 or with the value of the sign bit. If the variable is unsigned, the places will be filled with 0. The bits that shift out of the field are lost.

Suppose we have the following variables:

```
char sand = 8, shiftsand;
```

The following statement will store in `shiftsand` the value of `sand` shifted right two places.

```
shiftsand = sand >> 2;
```

Let's see what is happening behind the scenes with the value in `sand`.

Before shift	0	0	0	0	1	0	0	0
After shift	0	0	0	0	0	0	1	0

As before, `sand` itself is not being shifted. The variable `shiftsand` is being set to the value of `sand` shifted right two places. If we wanted to shift `sand` itself, we could use the combined assignment version of the right shift operator.

```
sand >>= 2;
```

Shifting a number right by n places is the same as dividing it by 2^n (as long as the number is not negative). So, the example is the same as

```
sand /= 4;
```

The bitwise shift will almost always work faster, however.

Bit Fields

C++ allows you to create data structures that use bits as individual variables. Bit fields must be declared as part of a structure. Here is an example.

```
struct {
    unsigned field1 : 1;
    unsigned field2 : 1;
    unsigned field3 : 1;
    unsigned field4 : 1;
    }  fourbits;
```

The variable `fourbits` contains four bit fields: `field1`, `field2`, `field3`, and `field4`. Following the colon after each name is a number that tells how many bits each field should be made up of. In this example, each field is 1 bit in size. This structure is stored in memory in a regular `unsigned int`. Since we are only using four bits, the remaining ones will go unused.

Values may be assigned to the fields just as if it were a regular structure. In this example, we assign the value 1 to the `field1` member:

```
fourbits.field1 = 1;
```

Since these fields are only 1 bit in size, we can only put a 1 or a 0 in them. We can expand the capacity of bit fields by making them larger, as in the following example:

```
struct {
    unsigned field1 : 1;
    unsigned field2 : 2;
    unsigned field3 : 3;
    unsigned field4 : 4;
    }  mybits;
```

Here, `mybits.field1` is only 1 bit in size, but others are larger. `mybits.field2` occupies 2 bits, `mybits.field3` occupies 3 bits, and `mybits.field4` occupies 4 bits. Here is a table that shows the maximum values of each field:

Field Name	Number of Bits	Maximum Value
mybits.field1	1	1
mybits.field2	2	3
mybits.field3	3	7
mybits.field4	4	15

This data structure uses a total of 10 bits. If you create a bit field structure that uses more bits than will fit in an `int`, the next `int` sized area will be used. Suppose we declare the following bit field structure on a system that has 16 bit integers.

```
struct {
   unsigned tiny    : 1;
   unsigned small   : 4;
   unsigned big     : 6;
   unsigned bigger  : 8;
   unsigned biggest : 9;
   } flags;
```

The problem that occurs here is that `flags.bigger` will straddle the boundary between the first and second integer area. The compiler won't allow this to happen. `flags.tiny`, `flags.small`, and `flags.big` will occupy the first integer area, and `flags.bigger` will reside in the second integer area. There will be five unused bits in the first. Likewise, since `flags.bigger` and `flags.biggest` cannot fit within one integer area, `flags.biggest` will reside in a third area. There will be eight unused bits in the second area, and seven unused bits in the third.

You can force a field to be aligned with the next integer area by putting an unnamed bit field with a length of 0 before it. Here is an example:

```
struct {
   unsigned first  :  1;
                   :  0;
   unsigned second :  1;
                   :  0;
   unsigned third  :  2;
   } scattered;
```

The unnamed fields with the 0 width force `scattered.second` and `scattered.third` to be aligned with the next `int` area.

You can create unnamed fields with lengths other than 0. This way you can force gaps to exist at certain places. Here is an example.

```
struct {
   unsigned first  :  1;
                   :  2;
   unsigned second :  1;
                   :  3;
   unsigned third  :  2;
   } gaps;
```

This will cause a 2-bit gap to come between `gaps.first` and `gaps.second`, and a 3-bit gap to come between `gaps.second` and `gaps.third`.

Bit fields are not very portable when the physical order of the fields and the exact location of the boundaries are used. Some machines order the bit fields from left to right, but others order them from right to left.