

Story 8. Rolling Around in Vegas



Recently the swaggering burly billionaire, Mr. Bones, introduced an exciting new dice game at his glitzy High-Rollin' Bones' Hotel and Casino. An oversized gold bowl containing four dice is presented to the player. The player inspects each die, removes whichever die seems the luckiest, and throws a \$100 chip in the bowl. Then Mr. Bones chooses one of the three remaining dice, takes a \$100 chip (showing his likeness) from his personal collection, and modestly places it into the bowl. Next the player and Mr. Bones roll their respective dice. Whoever rolls the higher number wins the two chips. Simple.

To make the game interesting, the four dice are not the run-of-the-mill dice we remember from the gambling-free days of our youth. While each die does have six sides as usual, their faces are marked in unusual ways. The kit that accompanies each new copy of *The Heart of Mathematics: An invitation to effective thinking* contains these four special dice. Roll 'em on out of your kit.

One die has two 6's and four 2's. Another has three 5's and three 1's. The third has four 4's and two blank faces. The last die has 3's on each face. The dice are not weighted—that is, any face is just as likely to land face-up as any other.

Deep Pockets Drew strides up to the bowl to choose the winning die. Which die should Drew draw? Drew considers the die that has all 3's. Which die could Mr. Bones select that will beat the all-3's die two-thirds of the time? After finding that die, we know that the all-3's die would not be a particularly wise choice.

Next Deep Pockets Drew considers the die with four 4's and two blank faces. Why will the die with three 5's and three 1's beat it two-thirds of the time? After verifying this dicey dominance, we know that selecting the die with four 4's and two 0's would not be a smart move.

Drew next considers the die with three 5's and three 1's. Why will the die with two 6's and four 2's beat it two-thirds of the time? After confirming this superiority, we know that the die with three 5's and three 1's would not be the best die.

Only one possibility remains: the die with two 6's and four 2's. Is there a die that will beat it two-thirds of the time? Your surprising discovery will show that none of the four dice is the "best" one to select, because each one can be beaten by one of the other three dice two-thirds of the time. Amazing.

So now Drew can put the dice in a circular order where each one beats its clockwise neighbor two-thirds of the time. What is that order? After doing the math, Deep Pockets Drew chooses not to play, and as a result his pockets become deeper.

This intriguing dice game surprisingly leads to the seemingly unrelated insight that the idea of a fair and democratic voting system is impossible—so much for "a government of the people, by the people, and for the people."

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Nudges:

- To compare two dice, consider making a chart with the columns labeled by the six numbers on one die (some of them duplicates) and the rows labeled by the six numbers on the other die (some of them duplicates).
- Then put the corresponding numbers in each square and see in how many square one die beats the other.
- If one die wins 24 of the 36 times, then that die will win two-thirds of the time.

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Nudges:

- A chart for the die with three 5's and three 1's versus the die with four 4's and two blanks has been started for you.

	1	1	1	5	5	5
0	(0,1)			(0,5)		
0	(0,1)			(0,5)		
4	(4,1)			(4,5)		
4	(4,1)			(4,5)		
4	(4,1)			(4,5)		
4	(4,1)			(4,5)		