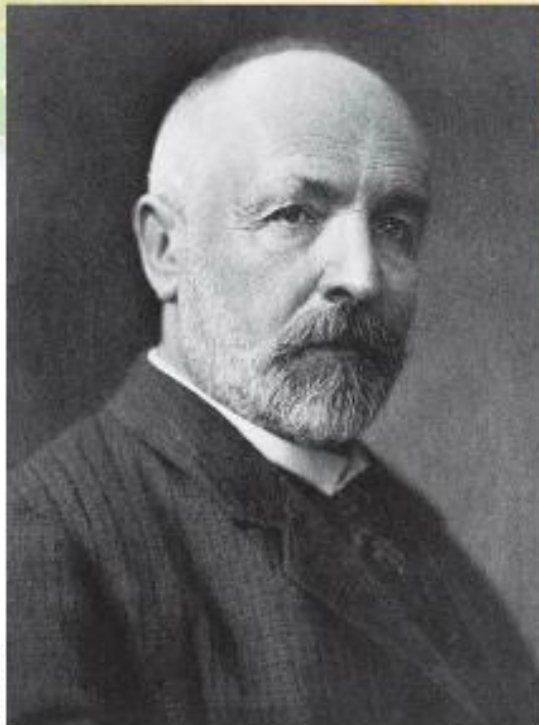


## 3.3

# THE MISSING MEMBER

*Georg Cantor Answers: Are Some Infinities Larger than Others?*



© Corbis

Georg Cantor, the first person to tame infinity.

*From the paradise created for us  
by Cantor, no one will drive us out.*

DAVID HILBERT

## QUESTION OF THE DAY:

Is there an infinite set  
“bigger” than the Natural  
Numbers?



# Cardinality

- The cardinality of a set  $A$  is the number of elements in  $A$ , which is written as:
  
- Examples of cardinality:
  - $|\{3,4\}| =$
  - $|\{5,6,5,6,5,6,5,1,1,1\}| =$
  - $|\{ \}| =$



# Quantitative Literacy Lottery



- **Lottery General Game Rules:**

- You must be enrolled in Quantitative Literacy to play.
- Players are responsible for completing their own tickets.
- A ticket is void if stolen, unissued, mutilated, **illegible**, tampered with or altered in any way, defective or incomplete.
- Your instructor is not responsible for lost or stolen tickets.
- You must fill in your name, address and phone number on the back of your winning ticket before you file a prize claim.
- All winning tickets are subject to validation by the Lottery.
- All prizes must be claimed within 6 minutes from the date and time of drawing.
- When two or more people win the lottery at the same time, the lottery commission has the mandate to split the prize, such that each winner gets an equal amount of the prize.

# Quantitative Literacy Lottery



- **How to Play the Quantitative Literacy Lottery**
  - Your first loto ticket is included in the price of your class.
  - Each additional wager costs \$  $\pi^{\varphi^{\pi}}$ .
  - For each wager, select four numbers from a pool of 1 to 20. (Do not repeat a number.)
  - Place each number in the desired order on your lotto ticket. (Yes, order does matter!!)
  - Check your loto ticket carefully to make sure you have correctly marked the numbers you want, in the order you want.
  - Once a ticket has been marked, it cannot be cancelled.
  - ALL SALES ARE FINAL.

# And the Winner gets...



- Good Luck and let's play QL Lottery!!!
- What buying strategy would guarantee winning?

# Dodge Ball

Player One's game board

1						
2						
3						
4						
5						
6						

Player Two's game board

1	2	3	4	5	6

# Dodge Ball - Revised

Player One can list as many rows as he/she wishes as the first play of the game. What will guarantee a win for Player One?

Player One's game board

1						
2						
3						
4						
5						
6						

Player Two's game board

1	2	3	4	5	6

# Infinite Dodge Ball

Player One's game board

1						
2						
3						
4						
5						
6						

Player Two's game board

1	2	3	4	5	6

# Cantor's Theorem

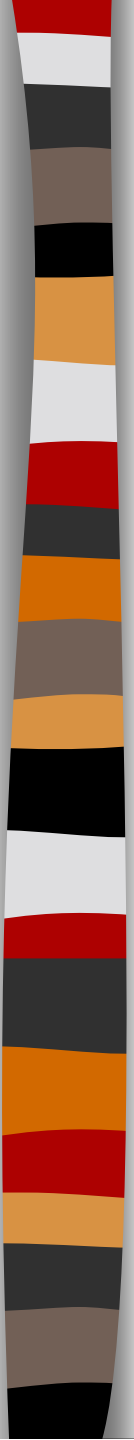
- There are more real numbers than natural numbers.



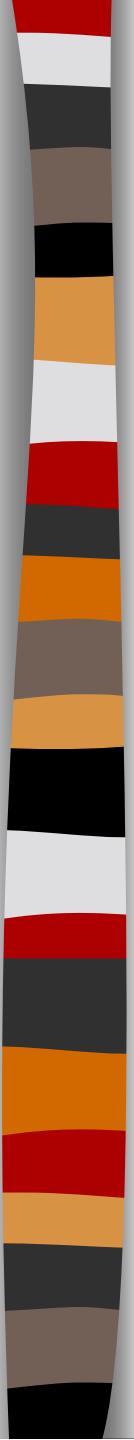


# Cantor's Theorem

- There are more real numbers than natural numbers.







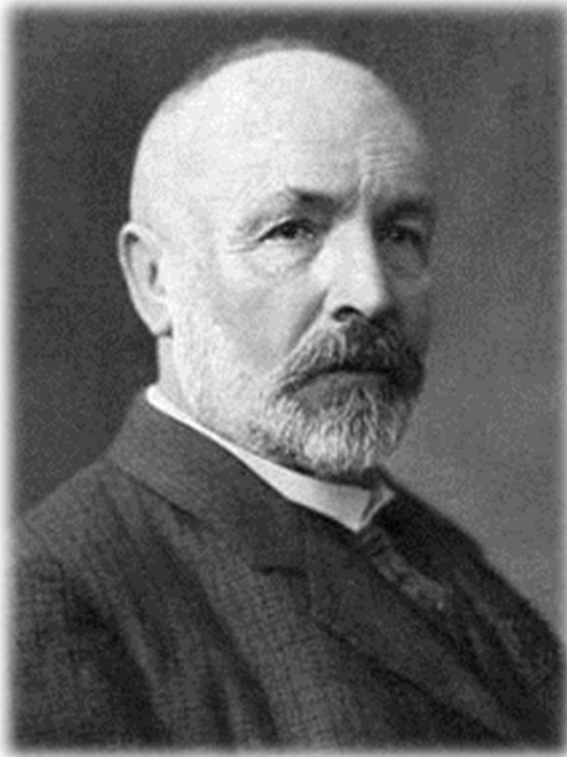
Natural Numbers		Real Numbers
1	←————→	0.55627363495617384921348...
2	←————→	142.02732981638472734718734...
3	←————→	7.61235987364823519197234...
4	←————→	238.18521936478912092519027...
5	←————→	-0.00083738265191836548713...
6	←————→	31.84722235675444566903346...
7	←————→	658.333333333333543356708632...
8	←————→	-37.83958382139857446882891...
⋮	⋮	⋮
11	←————→	29.99907982742111199853769...
⋮	⋮	⋮

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- If the number is a 2, replace with a 4.
- If the number is not a 2, replace with a 2.

$$M =$$

# Georg Cantor



- German Mathematician
- Born: March 3, 1845
- Died: January 6, 1918

- Founded set theory & introduced the concept of infinite numbers with his discovery of cardinal numbers.
- The entire mathematical community of his time struggled with his ideas about infinite sets.
- Near the end of the 19<sup>th</sup> century, many mathematicians resisted and attacked Cantor for his ideas.
- These attacks, along with other psychological issues, contributed to Cantor spending much of the last part of his life in an insane asylum.



# Homework

- Read 3.3 The Missing Member pgs. 163-173.
- Mindscapes 3.3 #4, 9, 11, 13, 14, 16, 17, 19
- Famous Mathematician
- The Math Book Presentation