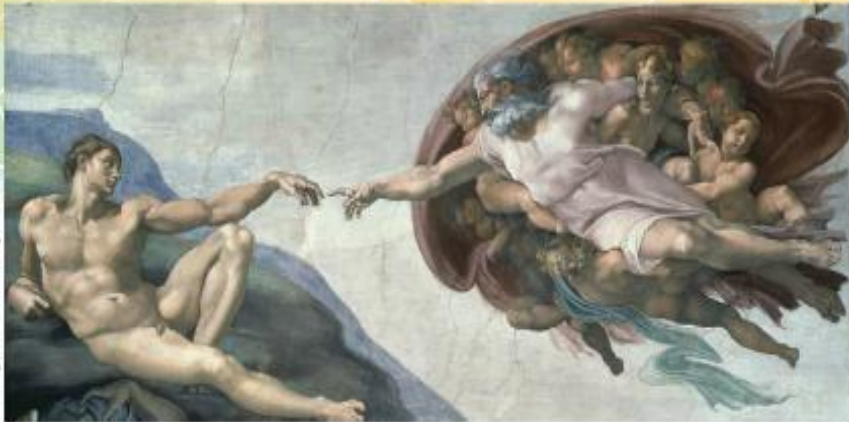


## 3.2

# COMPARING THE INFINITE

*Pairing Up Collections via a One-to-One Correspondence*



© SuperStock/SuperStock

Michelangelo Buonarroti, *The Creation of Adam* (1508–1512). An early one-to-one correspondence.

## QUESTION OF THE DAY:

How much bigger is the set of rational numbers than the set of natural numbers?

How many fractions are there between 0 and 1?



# Finite & Infinite Sets

- Finite set –
  - What is the smallest finite set?
- Infinite set -

# Bridge to Infinity

- Which set is larger, the Natural Numbers or the Natural Numbers with the 1 deleted?

Natural Numbers	
A Pairing	
Our New Set	

Natural Numbers	
A Pairing	
Our New Set	

# Cardinality

- Cardinality –
- Two sets have the same cardinality if



How would you show a one-to-one correspondence between the set of stars and the set of states?



# Cardinality of Other Sets...

- Compare the cardinality of the even natural numbers with the cardinality of the natural numbers.



# Seeking Higher Cardinalities

- Is there a set of numbers that has a greater cardinality than the cardinality of the natural numbers?



# The Integers and Natural Numbers

- Do the integers and the natural numbers have the same cardinalities?

# The Integers and Natural Numbers

Natural Numbers		All Integers
1	←————→	0
2	←————→	1
3	←————→	-1
4	←————→	2
5	←————→	-2
6	←————→	3
7	←————→	-3
⋮	⋮	⋮
$2n$	←————→	$n$
(The even numbers)		
$2n + 1$	←————→	$-n$
(The odd numbers)		
⋮	⋮	⋮



# Rational Numbers

- Create a list of rationals:

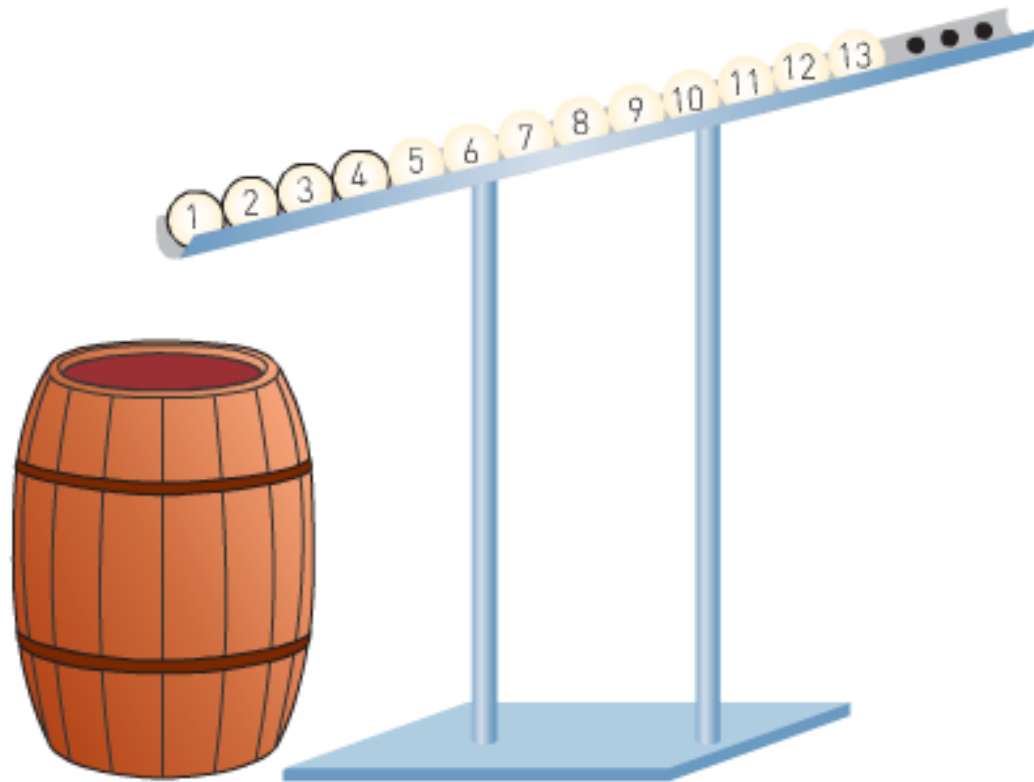


# Rational Numbers

- Can you construct a one-to-one correspondence between the rationals and the natural numbers?

# The Ping-Pong Ball Conundrum

- Read pages 150-151



# The Ping-Pong Ball Conundrum

Task Number	Time Left on the Clock at End of Task	Balls Added to the Barrel	Balls Removed from the Barrel

**How many balls are left in the barrel at the end of one minute?**



# Homework

- Read 3.2 Comparing the Infinite pgs. 147-162.
- Mindscapes 3.2 #3, 14, 16, 30, 32, 36
- Famous Mathematician
- The Math Book Presentation