

## 2.3

# PRIME CUTS OF NUMBERS

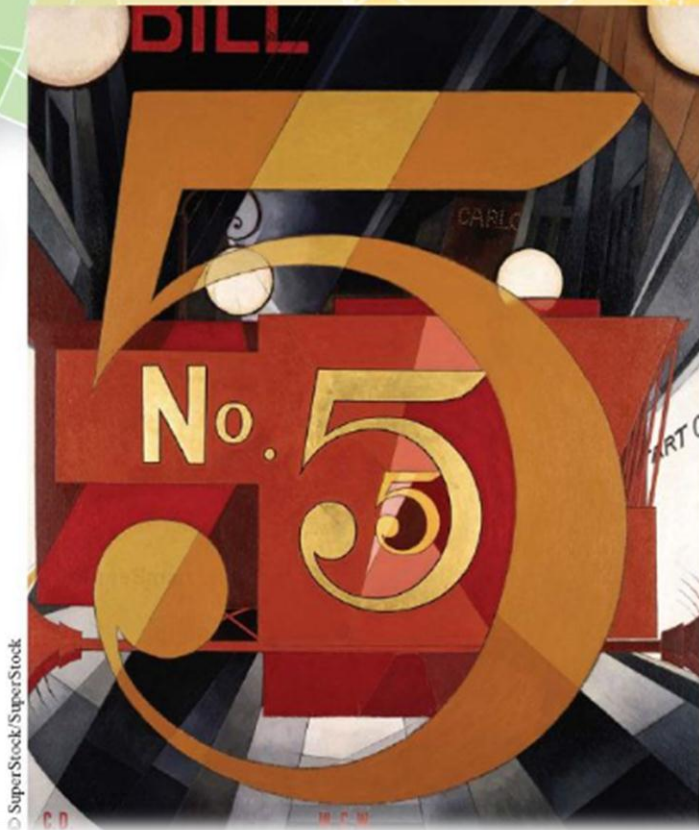
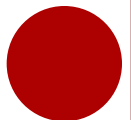
*How the Prime Numbers Are the Building Blocks  
of All Natural Numbers*

*... number is merely the  
product of our mind.*

KARL FRIEDRICH GAUSS

## QUESTION OF THE DAY:

Can you write 71 as  
a product of two  
smaller numbers?





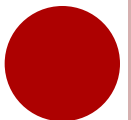
# WRITE AS A PRODUCT

- Write the following numbers as a product of smaller numbers, other than 1.

**12**

**21**

**36**





# DIVISIBILITY RULES

*A number is divisible by:*

- 2 if:
- 3 if:
- 4 if:
- 5 if:
- 6 if:
- 7:
- 8 if:
- 9 if:
- 10 if:



# THE PRIME FACTORIZATION OF NATURAL NUMBERS

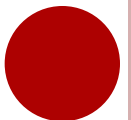
- Every natural number greater than 1 is either a prime number or it can be expressed as a product of prime numbers.
- Write the following numbers as products of prime numbers.

**12**

**215**

**360**

**210**

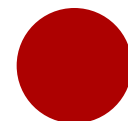


# REALLY BIG NUMBERS: WE CAN, BUT WE CAN'T

- Factor this long number:

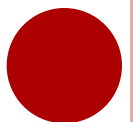
220301987135109256019257341093471092387102483561897  
265827364587640587619845691832465019238470192374187  
265018263541098327401923847019234601982561092384760  
198237401923867450872364098236740198236019238609236  
401928347012983470129348710293861023984610398710239  
8471092387102938710239871923871093

- The fastest computers in the world working full time would take centuries to factor this number.
- Our inability to factor really big numbers is the key to devising public key codes for sending private information over the Web and using e-mail and ATMs.



# PRIME NUMBERS UP TO 100

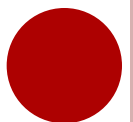
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100



# PRIME NUMBERS

- The first 100 prime numbers are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, 101, 103, 107, 109, 113, 127, 131, 137, 139, 149, 151, 157, 163, 167, 173, 179, 181, 191, 193, 197, 199, 211, 223, 227, 229, 233, 239, 241, 251, 257, 263, 269, 271, 277, 281, 283, 293, 307, 311, 313, 317, 331, 337, 347, 349, 353, 359, 367, 373, 379, 383, 389, 397, 401, 409, 419, 421, 431, 433, 439, 443, 449, 457, 461, 463, 467, 479, 487, 491, 499, 503, 509, 521, 523, and 541.

○ <http://primes.utm.edu/lists/small/10000.txt>

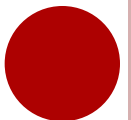


# THE INFINITUDE OF PRIMES

- Are there infinitely many prime numbers?  
Why or why not?



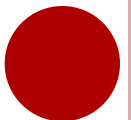
- **There are infinitely many prime numbers.**



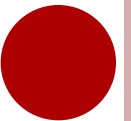
# THE INFINITUDE OF PRIMES – JUSTUS VAN GENT, EUCLID



- For each and every given natural number, we can always find a prime number that is larger than that natural number.
  - Let  $m$  represent an arbitrary natural number. Our goal is to show that there is a prime number that exceeds  $m$ .
  - Let  $N$  represent 1 plus the product of all natural numbers from 1 to  $m$ :

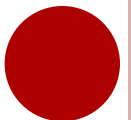


# THE INFINITUDE OF PRIMES - EUCLID



# THE INFINITUDE OF PRIMES - EUCLID

- Find a number bigger than 4 that is not evenly divisible by 2;
- Find a number bigger than 4 that is not evenly divisible by 3; and
- Find a number bigger than 4 that is not evenly divisible by 4.



# THE INFINITUDE OF PRIMES – EUCLID

- Suppose we only know one prime number = 2.

$$p_1 = 2$$

- Using the notation in the proof:

$$N = 2 + 1 = 3$$

- $N$  is prime so we add  $N$  to our list of primes 2, 3

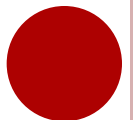
$$N = 2 \times 3 + 1 = 7$$

- $N$  is prime so we add  $N$  to our list of primes 2, 3, 7

$$N = 2 \times 3 \times 7 + 1 = 43$$

- $N$  is prime so we add  $N$  to our list of primes 2, 3, 7, 43

$$N = 2 \times 3 \times 7 \times 43 + 1 = 1807 = 13 \times 139$$



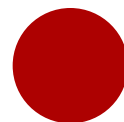
# FERMAT'S LAST THEOREM

- Read The Mostly True Story – pg. 77
- It is impossible to write a cube as a sum of two cubes, a fourth power as a sum of two fourth powers, and, in general, any power beyond the second as a sum of two similar powers.

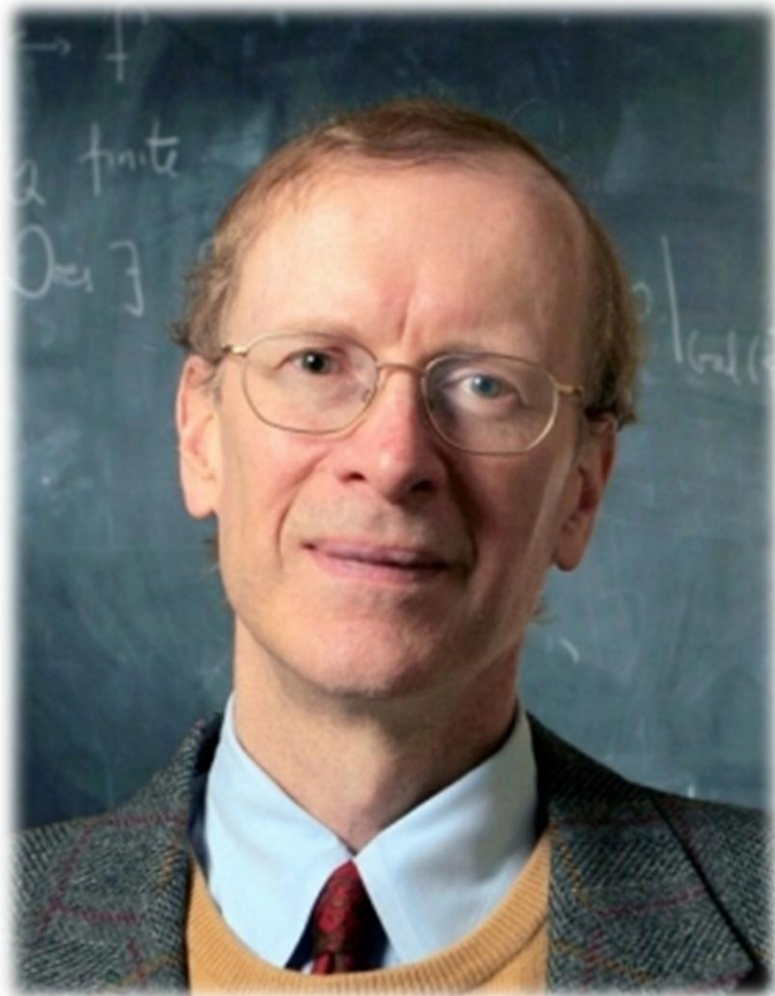


If  $n > 2$ ,

$$x^n + y^n \neq z^n.$$



# FERMAT'S LAST THEOREM (COMPLETE)



Andrew Wiles

*1994 - Over 130 page proof!!!*

$$x^2 + y^2 = z^2$$

<http://www.youtube.com/watch?v=7FnXgprKgSE>



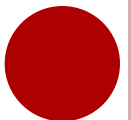
# THE TWIN PRIME QUESTION

- Are there infinitely many pairs of prime numbers that differ from one another by two?
- Examples:

11 and 13,

29 and 31,

41 and 43



# THE GOLDBACH QUESTION

- Can **every** positive, even number greater than 2 be written as the sum of two primes?
- Examples:

$$4 = 2 + 2$$

$$6 = 3 + 3$$

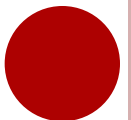
$$8 = 3 + 5$$

$$10 = 5 + 5$$

$$12 = 5 + 7$$

$$14 = ?$$

$$16 = ?$$



# HOMEWORK

- Read 2.3 Prime Cuts of Numbers – pgs. 68-85.
- List the first 20 Prime numbers.
- Write a one to two paragraph summary on the video **The Proof.**
- Mindscapes 2.3 #2, 7, 12, 14, 15, 24, 32, 35, 36
- A Million Things
- Famous Mathematician

