# 2.2 NUMERICAL PATTERNS IN NATURE

*Discovering the Beauty of the Fibonacci Numbers*

*There is no inquiry which is not finally reducible to a question of Numbers; for there is none which may not be conceived of as consisting in the determination of quantities by each other, according to certain relations.*

AUGUSTE COMTE

## QUESTION OF THE DAY:

**What is the next number in the sequence:**
**1, 1, 2, 3, 5, 8, 13, 21, ___**

*There can be great value*
*in looking at*
*simple things deeply,*
*finding a pattern,*
*and using the pattern*
*to gain new insights.*

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# Pineapples

- List as many observations about the pineapple as you can.

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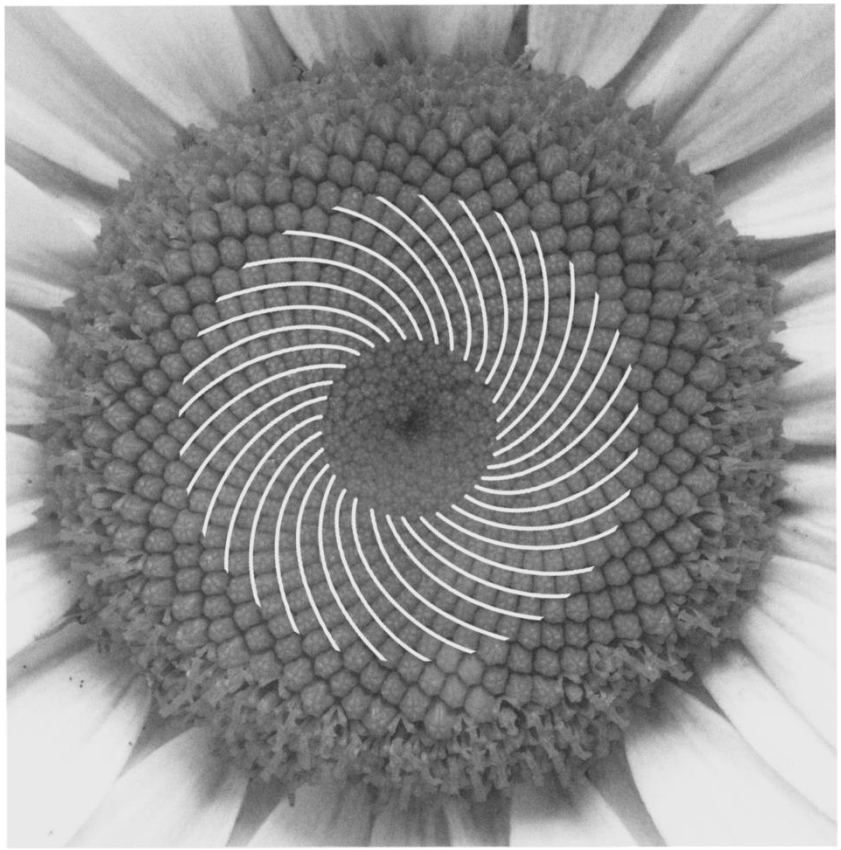
# The Daisy

Count the spirals in a daisy.

# The Daisy

Count the spirals in a daisy.



34 spirals

# Comparing Numbers

- The pineapple has two sets of spirals: 8, 13
- The daisy has two sets of spirals: 21, 34
- Compare these numbers: 8, 13, 21, 34

Do you notice a pattern?

# Noticing a pattern

- Find the next two numbers in the sequence:

8, 13, 21, 34, _____, _____

# More of the pattern…

- What numbers must have come before 8, and how many numbers before 8 exist?

____, ____, ____, ____, ____, 8, 13, 21, 34, 55, 89, …

# Fibonacci Numbers

- The following sequence of numbers are called the Fibonacci Numbers:

Leonardo of Pisa, or Fibonacci

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, …

https://www.khanacademy.org/math/vi-hart/v/doodling-in-math--spirals--fibonacci--and-being-a-plant--1-of-3

# Fibonacci Quotients

Find the Quotient of two Consecutive Fibonacci Numbers

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, …

| Fraction of Adjacent Fibonacci Numbers | Decimal Equivalent |
|---|---|
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |

# What number do we get?

As the Fibonacci Numbers in the previous quotients get larger and larger, what number are we approaching?

# Unending 1's

$\frac{1}{1} =$

$\frac{2}{1} =$

$\frac{3}{2} =$

$\frac{5}{3} =$

$\frac{8}{5} =$

# The Golden Ratio

$$\varphi = 1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \ddots}}}$$

# The Golden Ratio

$$\varphi = 1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \ddots}}}}$$

# The Golden Ratio

$$\varphi = 1 + \frac{1}{\varphi}.$$

Solve this equation for phi!

$$\varphi = 1 + \frac{1}{\varphi}$$

https://www.khanacademy.org/math/vi-hart/v/doodling-in-math-class--spirals--fibonacci--and-being-a-plant--2-of-3

# Express each non-Fibonacci Number as a sum of non-adjacent Fibonacci Numbers

1 = Fibonacci Number

2 = Fibonacci Number

3 = Fibonacci Number

4 =

5 = Fibonacci Number

6 =

7 =

8 = Fibonacci Number

9 =

# Express each non-Fibonacci Number as a sum of non-adjacent Fibonacci Numbers

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, …

| Natural Numbers | Sum of Fibonacci Numbers |
|---|---|
| 10 | |
| 11 | |
| 12 | |
| 13 | *Fibonacci Number* |
| 14 | |
| 15 | |
| 16 | |
| 17 | |
| 18 | |
| 19 | |

# Fibonacci Nim

Rules:

1) Start with a pile of sticks.
2) Person one removes any number of sticks (at least one but not all) away from the pile.
3) Person two removes as many as they wish with the restriction that they must take at least one stick but no more than two times the number of sticks the previous person took.
4) The player who takes the last stick wins.

# Homework

- Read 2.2 Numerical Patterns in Nature – pgs. 53-67.
- Mindscapes 2.2 #1, 2, 5, 6, 7, 17, 28, 30, 36
- A Million Things
- Famous Mathematician

# Mindscape 6 – Baby Bunnies

- Suppose we have a pair of baby rabbits: one male and one female. Let us assume that rabbits cannot reproduce until they are one month old and that they have a one-month gestation period. Once they start reproducing, they produce a pair of bunnies each month (one of each sex). Assuming that no pair ever dies, how many pairs of rabbits will exist in a particular month?

Start 

Month 1 

Month 2 

Month 3 

Month 4 

# Fibonacci Numbers – Baby Bunnies

Start 

Month 1 

Month 2 

Month 3 

Month 4 

| Time in Months | Start | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| Number of Pairs | | | | | | | | |